Gesture Authentication for Smartphones: Evaluation of Gesture Password Selection Policies

Eunyong Cheon¹, Yonghwan Shin¹, Jun Ho Huh², Hyoungshick Kim³ and Ian Oakley¹

¹Department of Human Factors Engineering, UNIST, Republic of Korea

²Samsung Research, Seoul, Republic of Korea

³Department of Software, Sungkyunkwan University, Republic of Korea

Email: beer@unist.ac.kr, yonghwanshin@unist.ac.kr, junho.huh@samsung.com, hyoung@skku.edu, ian.r.oakley@gmail.com

Abstract-Touchscreen gestures are attracting research attention as an authentication method. While studies have showcased their usability, it has proven more complex to determine, let alone enhance, their security. Problems stem both from the small scale of current data sets and the fact that gestures are matched imprecisely - by a distance metric. This makes it challenging to assess entropy with traditional algorithms. To address these problems, we captured a large set of gesture passwords (N=2594) from crowd workers, and developed a security assessment framework that can calculate partial guessing entropy estimates, and generate dictionaries that crack 23.13% or more gestures in online attacks (within 20 guesses). To improve the entropy of gesture passwords, we designed novel blacklist and lexical policies to, respectively, restrict and inspire gesture creation. We close by validating both our security assessment framework and policies in a new crowd-sourced study (N=4000). Our blacklists increase entropy and resistance to dictionary based guessing attacks.

I. INTRODUCTION

Smartphone password schemes like screen lock patterns and PINs suffer from the security issues that emerge from the biased ways in which users choose their passwords: they select easy-to-remember and quick-to-draw lock patterns [1] and PINs [2] that are also easy to guess. Free-form gestures are a potentially fast and secure way of authenticating users to smartphones that may address the problem. Rather than being constrained by a fixed grid or keypad layout and small password space (e.g., 389,112 possible lock patterns), users can freely draw one or more strokes on a touchscreen [3] or bespoke input surface [4]. The resultant stream of coordinates can be matched against stored templates, using mature algorithms [5], to grant or restrict access. The use of gestures conveys numerous advantages: the theoretical number of possible gestures is extremely large [3] and; gesture input may require less visual attention than input based on selecting buttons or targets [6] making it particularly suitable for mobile or wearable scenarios where users may be working on small displays [7] or busy and engaged in other dominant tasks [8].

While the potential benefits of gesture passwords are wellestablished, their applicability for use as a smartphone unlock scheme and their inherent security and usability remain unclear. One issue is the small size of existing data sets – prior work has collected between 22 [9] and 345 [10] different predefined or user-proposed gestures from 34 to 45 users and the largest analysis to date [11] combines two proprietary data sets, captured in different studies/settings, to yield a corpus of 529 different gestures in total. This contrasts to existing analyses of passwords [12], PINs [13] and pattern locks [14] which based security assessments on data sets of several thousand examples. In light of these precedents, we argue it is necessary to complement existing claims about the high entropy of gesture passwords, based on controlled small scale studies in the lab or field, with the data and analysis from a larger online study of gesture passwords.

A related problem is a lack of established metrics for assessing security - unlike the exact comparisons possible with passwords and PINs, gestures are matched via a similarity measure (e.g., cosine [15] or Dynamic Time Warping (DTW) [16] distance). There are currently no methods for establishing common security metrics such as guessing entropy for gesture data sets, making it hard to contrast gestures against other authentication methods. Researchers have instead made security assessments based on, for example, calculations of Equal Error Rate (EER) [9], [3] or via resistance to manual [3], [17] or brute-force guessing attacks [11]. Improving the quality and scale of gesture data sets is an integral current requirement for work in this area; doing so will enable development of new forms of security analysis, provide a more comprehensive assessment of the security of the basic technique, and provide raw evidence required to determine the suitability of gesture passwords as a lock scheme for smartphones.

This paper addresses these limitations and evaluates the feasibility and practicality of using gesture passwords as a main authentication technique on smartphones. In this scenario, we constrain gestures to involve single strokes by single fingers on a small screen region, similar to graphical pattern locks [4]. From this starting point, we describe a multi-stage study. We first capture the largest extant sample of gesture passwords (N=2594) from crowd workers. We analyze this to extract key security metrics, such as an EER for the gesture matching threshold, and as the basis for developing both a dictionary of the 20 most common gestures (Android only allows 20 consecutive fail attempts), and an automated entropy assessment algorithm. We show the dictionary is effective at cracking the gesture passwords: 54.18% to 58.37% at the EER threshold value and 23.13% to 31.49% with a stricter threshold derived from closely related prior work [10]. This indicates userchosen gesture passwords, just like other unlock schemes, are insecure against dictionary-based password guessing attacks.

To help users select stronger gesture passwords, we introduce three novel policies: a lexical policy that involves presenting words that can inspire gestures, and two blacklist policy variants that block users from choosing password gestures that match popularly used gestures. We capture and analyze a new crowd-sourced data set (N=4000) and compare the usability and security of gestures generated with these policies against those generated in a standard condition. The results show that while our lexical policy is ineffective, our blacklist policies increase both the entropy of gesture passwords and their resistance to future dictionary based guessing attacks: cracking as few as 14.93% of gestures when a consolidated blacklist policy was enforced. Usability trade-offs were small compared to the baseline policy: between 1.1 and 1.7 seconds slower in mean authentication times, and approximately 3% lower in 1-day recall tests (memorability).

The contributions are: 1) the largest data set of gesture passwords – over both studies 6594 unique participants produce more than 67 thousand examples of 9188 different gesture passwords. 2) a novel security assessment framework for gesture-based authentication that can generate a dictionary of representative gesture passwords, measure cracking rates, and calculate the entropy of user-chosen gesture passwords. 3) novel *blacklist* and *lexical* policies for improving the entropy of the gesture passwords users create. Finally, 4) an assessment of these policies in terms of both usability (over multiple recall sessions) and our newly developed security metrics.

II. RELATED WORK

The use of gestures to authenticate an individual has a long history; a written signature is, fundamentally, a 2D gesture. With the emergence of large, high resolution touch-screens on mobile devices, researchers began to explore how to apply gesture authentication to this new space. Early work focused on the biometric properties of gesture input, suggesting that how users stroke the screen while producing single [18] or multi-touch [9] gestures can provide identifying information with an accuracy of between 77% - 90% for a single input.

A. Gesture Passwords

More recent work has focused on gestures as passwords in and of themselves - when the strokes made, rather than how they are performed, is the data examined to authenticate a user [3]. Various aspects of performance have been examined in this space. One prominent strand of work has focused on gestures that contain one or more finger strokes, with the requirement that multiple finger-strokes overlap temporally, and examined optimal algorithms to recognize gesture passwords [5] and the usability and memorability of gesture passwords compared to text passwords after periods of one hour, one day and one week [10]. The authors conclude that gestures passwords achieve levels of usability equal to or exceeding text passwords - for a set of 91 participants creating and recalling pairs of gesture passwords, mean creation times of 69 seconds, recall rates of 89.6% and recall times of 16.49 seconds were indistinguishable from text password performance. We also

note that current touch screen gesture authentication work focuses on input over the whole screen; no existing work studies the use of gesture passwords on smartphones with a more limited drawing canvas. We focus on the feasibility of using gesture passwords as an alternative graphical password (lock) scheme on smartphones – as such we restrict input to the small phone screen regions typical in phone lock systems.

B. Gesture Password Security

Although the usability of gesture authentication schemes can be assessed in much the same way as any other form of password, assessing their security is more difficult. This is fundamentally because, unlike traditional password systems, matches between stored gestures and those entered by users (or attackers) is achieved not through an exact and precise comparison but by exceeding a threshold value on a similarity measure such as cosine distance (used in the Protractor recognizer [15]) or DTW distance [18], [16]. The key consequence of this approach is that there are many possible valid variations of a gesture that would authenticate a user and that different matching algorithms [5] and match thresholds (or number of template gestures [3]) will yield different levels of performance in terms of the proportion of genuine or malicious gestures that are confirmed as a valid match. In practice, this means that while a naïve analysis of the entropy of gesture passwords, defined as the number of possible strokes that can be made on a canvas of some given resolution, is extremely high (e.g., 100 bits for 16 point gestures drawn on an 8 by 9 grid [10]), the practical entropy of the space is likely much lower, as multiple strokes within this space will match one another. Unsurprisingly, as with other forms of password, users are biased to produce specific gestures more frequently than others [11].

Reflecting these problems, prior research assesses the security of gesture authentication schemes through alternative approaches. A common one is empirical: gesture passwords are attacked by either observers [3], [17], or via automated processes such as through random geometric guesses [5] and their resistance to these attacks is contrasted against data for baseline cases such as passwords. Recent automated attacks exploit information extracted from gesture data sets to improve performance: the symmetry of gestures and a dictionary of commonly selected gesture passwords [11]. Although it is not applicable to an online attack scenario (with guessing attempt limits), this attack was shown to be highly successful if used for an offline attack - crack rates were between 47.71% and 55.9% with 10^9 guesses. We note that the large sets of gesture password samples that would be necessary to generate and validate attack dictionaries do not currently exist.

C. Password Policies

Password selection policies, such as mandated minimum length or required special characters, can help users create stronger passwords [19]. However, policies that are effective at improving password entropy can negatively impact usability [20]. System-assisted password selection policies [14] can be effective. They try to guide users to create more secure passwords – significantly improving security with small compromises in recall rates and unlock times. Currently, only limited work has examined password selection policies for gesture passwords – Clark *et al.* [21] proposed three policies that request users to create gestures that are fast, random or use multiple fingers. Evaluations indicate they had limited impact on security and may have negatively affected usability. Given the importance of password system, we identify the development of policies that can help users select more secure gesture passwords as an underdeveloped area of research.

III. SECURITY EVALUATION FRAMEWORK

Here we discuss the challenges in measuring the security of gesture passwords, and propose a novel evaluation framework for gesture password security. Our framework consists of the following three methods: (1) configuring gesture algorithm parameters based on EERs, and measuring false acceptance rates as the first security measure; (2) measuring entropy of gesture passwords through an *n*-gram Markov Model; and (3) measuring the resistance of gestures to a novel clustering-based dictionary attack. This framework is used later to compare the security of our gesture selection policies.

A. Online Attack Model for Smartphones

To mitigate online guessing attacks on smartphones, only k consecutive fail unlock attempts are allowed (e.g., k = 20 for Android and k = 10 for iOS). That is, after k unsuccessful attempts, an attacker can no longer try unlocking the target device. Thus, the attacker's goal is to unlock a target device within k guessing attempts. If the attacker has no information about the gesture password being used, the best attack strategy is to try the top k most commonly used gesture passwords first. If a gesture password data set is available, the attacker could use a data-driven approach to build this list of top k gesture passwords, and try them sequentially to unlock victims' devices. In the next section, we explain why building a guessing dictionary for gesture passwords is challenging.

B. Challenges in Evaluating Security

We identify two key problems with assessing the security of gesture passwords in response to online attacks. The first relates to modelling the gesture space – as the theoretical space is very large, it is an open question how to best identify common gesture forms. Prior work has proposed manual classification of gestures into broad categories (e.g. digits, letters, geometric shapes [11]) to support offline attacks. There is no prior work exploring automated approaches to this problem, such as *n*-gram models or Probabilistic Context-Free Grammars (PCFG) [22] capable of computing probability scores for all possible gestures, or clustering algorithms that group gestures in a set based on their similarity. Due to this lack, we believe that exploring mechanisms to automatically determine common gesture classes is an important first step to support online attacks on gesture authentication systems.

The second relates to the fact that multiple possible variations of a gesture, effectively multiple different gestures, will authenticate any given user. This is because thresholded distance metrics, as opposed to exact similarity, are used to determine matches between entered gestures and stored templates. To create maximally effective dictionaries for online attack, it is therefore important to be able to generate or select highly representative gesture exemplars from common gesture classes. Prior online approaches to this problem [11], focused on making large numbers of diverse guesses by distorting randomly selected gesture examples, do not apply to an online attack scenario in which the maximum number of guesses is low (e.g., 10 or 20). A viable online attack against gesture passwords must be able to identify optimal guesses for each gesture class. In the next sections, we explain how our gesturetailored *n*-gram model and clustering-based dictionary have been designed to address these challenges.

C. Preprocessing and Equal Error Rates

Prior to performing recognition or other analysis, gestures need to be normalized. We follow recommendations from prior work [5] and apply scale and position normalization, making these proprieties effectively invariant – two gestures depicting similar leftward arrows should therefore be matched regardless of any differences in the scale or location of the strokes on the canvas. We do not apply rotation normalization, making gestures rotation variant – a leftward arrow would therefore not be matched to an otherwise similar rightward arrow.

There are many existing algorithms for gesture recognition. Our framework applies two recognizers that have been widely used in recent studies of gesture passwords: Protractor [3], [10] and Dynamic Time Warping (DTW) [11]. We describe the configuration of these algorithms below:

Protractor: We used the reference \$N Protractor implementation [23]. Gestures are compared by the inverse cosine distance between their vectors. We configured the recognizer to allow only single stroke gestures and to allow gestures to be matched on drawn shape rather than stroke sequence (i.e., to use original and inverted stroke sequences as templates). Rotation invariance was applied at the default thresholds: +- 30° degrees for the initial stroke, defined as 1/8 of the gesture length, and 45° for the gesture as a whole.

DTW: We used a standard DTW implementation based on a Euclidean distance measure [16]. No additional processes are required to maintain rotation variance.

A key final normalization of gestures is re-sampling: both protractor and DTW algorithms require that gestures being matched are the same size. They meet this constraint by resampling all strokes to a preset size; optimal values for this size parameter vary depending on the gesture set. As in prior work [11], we determine optimal values for this parameter by creating multiple sets of re-sampled gestures and examining Equal Error Rates (EERs). Specifically, we create 12 gesture sets with re-sampled lengths of between 8 and 96 points, in steps of 8. We calculate EERs by adapting processes described in Sherman *et al.* [3]. For a given data set, False Rejection Rates (FRRs), which measure the proportions of users' genuine gestures that are rejected by a gesture algorithm, are calculated by matching different examples of each individual's gestures against each other. False Acceptance Rates (FARs), which measure the proportion of others' gestures being misclassified as users' own, are based on matching an individual's stored gesture template against those from all other individuals. FARs reflect the rate at which attackers might succeed in guessing users' gestures. By calculating FRRs and FARs for a range of distance threshold values, we can derive an EER at their intersection. Re-sampling size is then set to minimize EERs across the data-sets being examined.

D. Entropy Analysis with n-gram Markov Model

Measuring the guessing entropy of passwords is commonly achieved by analyzing the probability distribution of realworld passwords. However, as we can only collect samples representing a small portion of the theoretically possible space of gesture passwords, we need to develop a probabilistic password model [22] that uses collected samples to estimate the probability distribution of all gesture passwords. We do this by developing n-gram Markov models that can calculate the probability of each gesture password. n-gram Markov models are an appropriate technique as they have successfully been used to estimate the probability distribution of other graphical password schemes [1], [14]. In an n-gram Markov model, the probability of the next stroke in a graphical password is calculated based on a prefix of length n. The idea is that adjacent strokes in user-chosen graphical passwords are not independent, but follow certain high probability patterns.

In order to produce an n-gram model, based on sequences of discrete tokens, from a continuous two-dimensional gesture representation we need to design a discretization process that minimizes error. Our multi-stage process is described below and illustrated in Figure 1.

Discretization: First, we apply Douglas-Peucker (DP) line simplification [24] to each gesture. We examine the relationship between the DP simplification tolerance value and the number of simplified strokes, selecting the knee point as the optimal value. Based on the resultant set of simplified strokes, we create discrete symbols based on stroke length and stroke angle. To identify a mapping that minimizes error, we consider multiple divisions of stroke length (dividing the full range of all observed stroke lengths into 2, 3 or 4 equally sized length regions) and angle (into 6, 8, 10, 12 and 14 equally sized angular regions). This leads to 15 differently sized models containing from between 12 (2 lengths by 6 angles) and 56 (4 lengths by 14 angles) possible symbols, roughly equivalent to the number of symbols in a pattern (9 points) or PIN (10 symbols) and an alphanumeric password (about 95 symbols). In addition, we also consider two possible phases for the angular regions: with an origin at 0° and an origin at half the region angular width (e.g., 22.5° if there are 8 regions). We refer to these phases as *aligned* and *offset*. This leads to a total of 30 different approaches. Following this transformation,

we are able to represent each gesture as a series of discrete symbols each representing a single stroke.

Generation: To estimate the probability of any possible gesture password from a set of gesture samples, it is essential to develop a probabilistic password model (e.g., ngram Markov model) that effectively represents the probability distribution of real-world gesture passwords. Therefore, we build a number of reasonable n-gram Markov models across various discretizations and *n*-gram Markov parameters. Using a 5-fold process, we create a set of nine 2-gram models using each of the 30 discretizations: 270 models in total. Each set explores a grid search over two additional variables: smoothing method ("add-1 Laplace smoothing", "add-1/(number of symbols) Laplace smoothing" and "Good-Turing smoothing") and; exclusion policy for short gestures. Specifically, we create models that 1) include all gestures, 2) exclude single stroke gestures, and 3) exclude single and *dual* stroke gestures. The use of smoothing methods enables n-gram models to cover rare n-gram cases. We explore excluding extremely short gestures as their probability may be over-weighted, potentially biasing the n-gram models. In each model, we apply end-point normalization to ensure the sum of probabilities of all possible gesture passwords is 1. We opt not to examine 3-gram models due to the difficulty of collecting a data set large enough to yield an acceptably low proportion of unseen cases.

Selection: To select reasonable models, we apply three criteria. The first two rely on comparisons between original userchosen gestures and gestures derived from the discretization process and n-gram Markov models. In order to make these comparisons, we reconstitute gestures from their symbolic forms in the models. This is done by creating a contiguous sequence of strokes, with each stroke's length and angle set to the central values of its length/angle segment - see steps 6 and 7 in Figure 1. The first criteria based on this process is the crack rate using k guesses at a specific distance threshold t. We believe models achieving a higher crack rate more accurately reflect the probability distribution of the collected samples. The second criteria is the similarity between each original user-chosen gesture and its discretized n-gram representation. For a given n-gram model, we calculate this as the proportion of gestures that are more similar than a specific distance threshold t. We believe models in which gesture representations are more similar to the originals will be more accurate. The third criteria we apply is the model completeness. We surmise that models in which we observe a larger proportion of possible n-gram cases will be more accurate. We calculate these values for all models. Models should be selected for further study through manual inspection to achieve a good balance of performance across these criteria.

Optimization: Any discretization method for *n*-gram Markov models yields degenerate cases (e.g., strokes on an edge). Thus, to improve *n*-gram Markov model performance, we need to handle degenerate cases in each selected model. We do this during discretization of strokes into length and angle categories by treating cases within b% of boundary edges as ambiguous and incrementing the *n*-gram sequence frequency



Fig. 1. Overview of discretization process from a gesture (step 1) to angle-region symbols for building *n*-gram Markov models (step 5) and the gesture reconstitution process for evaluating them (steps 6 and 7). Illustration uses a simple model with 2 length and 4 angle discretization regions.

of both the actual and adjacent length or angle region by i and j respectively, where the sum of these values is always 1. This reflects the fact that strokes near the discretization region boundaries may be erroneously (or noisily) classified. For each selected n-gram Markov model, we perform a grid search over values of b in the range 1% to 10% for angle and length with both i and j set to 0.5, thus generating an additional 100 n-gram models. The final model is selected based on balanced improvements to the metrics defined above.

Calculating partial guessing entropy: With the best performing *n*-gram Markov model(s), we calculate partial guessing entropy to evaluate the security of the gesture passwords. Partial guessing entropy estimates [25] are useful because real-world attackers might only be interested in cracking just a fraction of an entire password set. This is a popular technique for estimating the average number of trials needed to successfully crack a *fraction* (α) of an entire password set. We report these data in terms of "bits of information."

E. Clustering-based Dictionary Attack

To evaluate the security of gesture passwords against guessing attacks, we introduce a novel dictionary attack based on *clustering* to group gestures according to the similarity of their shape. The goal is to identify common gesture classes and, within each class, select the most representative gestures to support guessing attacks. We use a 5-fold process as follows:

Calculate distances: We calculate distances between all gestures in the training set using a gesture distance metric (e.g., Protractor or DTW). This data is identical to the FAR calculation described in Section III-C.

Cluster gestures and select representative examples: We apply the affinity propagation clustering algorithm [26] to these distances. The key advantage of affinity propagation over alternative approaches is that it is an exemplar-based clustering algorithm that identifies representative examples (in our case, typical gestures) within the data-set. We believe this can be useful for creating a dictionary of gesture passwords. The results of this algorithm are a set of clusters containing similar gestures. Each cluster has a specific gesture at its

geometric center. We argue this central gesture will be the optimal representative gesture for the cluster it is derived from.

Rank gestures and create dictionary: We order the clusters by size and create a dictionary of center gestures of the largest k clusters. We evaluate the fit of clustering model by examining the number of clusters generated and the *mean inter-cluster distance*, defined as the mean distance between all gestures in each cluster. A larger number of clusters likely reflects a more diverse gesture set. Similarly, clusters containing gestures that are more distant from one another can be assumed to contain gestures with more diverse shapes – center gestures may therefore be less representative of the full cluster contents.

Perform dictionary attack: With the dictionary of k center gestures, we match all test set gestures against the dictionary. The crack rate is the proportion of test set gestures that match at least one gesture in the dictionary. We report crack rates for a continuum of distance thresholds and/or corresponding FRRs. This serves as the primary metric for evaluating the security of a set of gesture passwords. If different conditions, models, algorithms or policies are being compared, the crack rates may also be tested for significant differences using contingency tests. Testing or other comparisons should take place at standardized FRR levels such as 2.5%, 5% or 10%, or those used in relevant prior literature.

IV. FIRST STUDY: GESTURE PASSWORD SECURITY

We designed, implemented and executed an on-line study to capture the largest set of password gestures to date. The gestures were captured in a homogeneous study protocol and outside traditional lab or university environments. The goals of capturing this data set were to (1) move beyond the small scale security analyses enabled by prior research and remove biases, such as the experimenter effect or a lab based population bias, from the data, (2) measure the security of gesture passwords using the proposed framework, (3) gauge the effectiveness of the framework, and (4) acquire a large set of gesture samples to build blacklist policies. The ethical aspects of the study were approved by the host university IRB.

A. Gesture Recognizer

During data capture, we used Protractor, configured as described in Section III-C, as it is mature and has been deployed in a number of closely related studies [3], [10]. Protractor's similarity measure is the inverse cosine distance: when two gestures are compared, larger scores indicate greater similarity. While data about specific thresholds used in prior gesture authentication systems is scarce, one prior study with Protractor uses a threshold of 2.0 [10]. Considering our goals, we opted to set Protractor's matching threshold to be more permissive: 1.0. We explicitly used this permissive threshold in order to capture a greater proportion of the raw gestures that participants produce; as our study protocol used gesture matching to ensure validity of the input gestures (see next section), stricter thresholds that increase the difficulty of matching gestures might serve to filter or restrict the gestures that participants were able to create during the study. A permissive threshold ensured participants entered meaningful gestures that resembled each other while minimally constraining the form and type of those gestures. This best met the goals of our study. We also note this is a typical approach in empirical work to capture gestures - rather than constrain examples to fit a given algorithm during collection, gathered examples are considered as valid gestures that can be studied to develop optimal recognition algorithms in subsequent offline analysis. In this way, the use of a permissive threshold allows capture of a valid but minimally restricted set of gestures that can support the broadest possible set of future analyses - by, for example, examining the impact of increasingly strict thresholds or applying different comparison algorithms.

B. User Study Design

The study was implemented as a website and participants were recruited via Amazon Mechanical Turk (MTurk). The MTurk listing summarized study activities, requested participants to complete it on a mobile device and provided both a link and a QR code to the study site. We screened participants who arrived at the study site as to whether they were on a mobile device or not – we checked for a touchscreen device with a portrait orientation, common criteria for browserbased mobile device detection. Participants not satisfying these criteria were reminded to complete the study on a mobile device and again provided with the study link and QR code.

Participants accessing the study site on a mobile device were first presented with instructions requesting they create a *passgesture* they would use to secure access to their mobile device. Two incentives were provided to encourage the creation of secure, memorable gestures. Firstly, we informed participants they would also create an *attack-gesture* that would be used to guess the gestures of other study participants; similarly, the attack-gestures of other study participants would be used to guess their own pass-gesture [1]. If these guesses were successful, they would not receive compensation. Secondly, we told participants they would be asked to recall their passgesture up to one day after creating it.



Fig. 2. The first study running on a Samsung Galaxy 8 Android mobile phone in Google Chrome. Figures show limited input region (following Android pattern lock) and pass-gesture creation after an example gesture has been drawn (a) and pass-gesture confirmation screens for gestures that match (b) and fail to match (c) the example gesture (a).

Participants then moved on to a screen that asked basic demographics (handedness and categories for age, educational level, occupation and ethnicity). They then progressed to a screen where they first created and, on a subsequent screen, confirmed their pass-gestures. Confirmation pass-gestures were required to match creation pass-gestures, using the Protractor recognizer configured as described in Section IV-A. A failure to match these gestures led to both being deleted and the passgesture set up process starting again. Participants were also able to cancel the set up process at any stage, resulting in a similar deletion of the templates and restart of the process. Once gestures had been matched successfully, and participants were satisfied, they tapped on a button to finalize their gestures and move on in the study. This interface was closely modeled on the pattern lock set up process on Android mobile phones and can be seen in the screen shots in Figure 2. We note the input region used was limited, following typical phone pattern lock and PIN implementations. This helped ensure the input region was accessible to users regardless of their physical size, handedness and phone grip - the area is reachable by the thumb of most users in a single handed grip, a practical constraint for any realistic phone unlock system.

The next stage of the study involved a similar process of gesture creation and then confirmation for the attackgesture. Additionally, attack-gestures were matched against pass-gesture templates; if matched, participants were informed attack-gestures needed to differ from pass-gestures and the attack-gesture creation process was restarted. The attack gesture served as a distracter task to clear each participants shortterm memory. Finally, participants completed a recall task for their pass-gesture. They had a maximum of five attempts to match either their creation or confirmation pass-gesture. The limit of five was derived from the Android security system after five incorrect entries, users need wait 30 seconds before making further attempts. After either a successful match or five failures, the study terminated with a screen informing participants of their correctness in the recall task, by thanking them for them time and with a numerical code that enabled them to register the study as complete on MTurk.

C. Measures

For each participant we recorded the following measures. All measures relating to gesture creation and confirmation were logged for both pass- and attack-gestures; measures for recall only relate to pass-gestures.

All gestures. We logged all entered gestures at all phases of the study. Gestures were tagged with the study stage and participant ID and recorded at the native resolution (both temporal and spatial) of the participant's device. Additionally, we tagged all final or correct creation, confirmation and recall gestures. If a user failed to recall their pass-gesture within five attempts, no correct recall gesture was recorded.

Setup time. We measured the time to create and confirm gestures from first presentation of the gesture creation screen through to when the confirmation gesture was accepted by the user via a explicit button press to move on with the study.

Setup cancels. We logged the number of times the gesture set up process was intentionally canceled by a user wishing to revise or change their gesture.

Setup failures. We stored the number of times that confirmation gestures failed to match creation gestures.

Recall rate. We logged the number of participants who failed to recall their pass-gesture within five attempts.

Recall attempts. We logged the number of recall failures regardless of final success or failure in the overall recall task. These failures were due to either matching one of the attack-gestures or failing to match any gesture.

Recall time. For those participants who successfully recalled their pass-gestures, we logged the recall time from the first presentation of the gesture entry screen through to successful entry of the gesture.

D. Participants

In total, 2619 unique Amazon Mechanical Turk workers completed the study, each rewarded with 0.25 USD. On manual inspection, we removed 25 participants from the set, as they created essentially identical (and highly unique) gestures in close temporal proximity – we assumed they were created by a single individual with access to a large number of separate MTurk accounts. The final gesture set thus contains data from 2594 separate MTurk workers. Participants completed the study, from initial gesture creation through attack gesture creation to final recall entry, in a median of 47 seconds, corresponding to a median hourly wage of 19 USD.

E. Usability Results

1) Demographics: The majority of participants identified as white (56.32%), Asian (18.47%), Hispanic (9.48%) or black / African-American (7.94%) and fell in the 18-24 (31.8%), 25-34 (46.72%) or 35-44 (15.42%) age groups. Most were educated post-graduate (13.61%), college (49.46%) or high school level (33.73%) and they worked in a wide range of fields; the largest group were students (15.38%).

2) Setup Cancels, Failures and Time: Data for setup of pass- and attack-gestures is shown in Table I. Due to positive skews in the majority of data we report means, SDs and medians for all measures. We note the 24.38 seconds setup time is substantially below figures in the literature of gesture passwords – for example Yang et al. [10] report a setup time of 69 seconds for their multi-finger gesture passwords. The most likely explanation for this variation lies in the different participant populations (MTurk workers vs lab study participants) and our focus on single finger, single stroke gestures over the more complex multi-finger or multi-stroke gestures studied by Yang et al. [10]. Although shorter than prior free-form password systems, these setup times remain representative of online studies of other graphical authentication schemes - in a recent crowd sourced study of pattern locks, mean setup times for a standard system are reported to be 22.05 seconds [14], broadly similar to data reported here. We interpret this to mean that users both appropriately engaged with the gesture creation task, and were also able to create gestures at reasonable speeds.

3) Recall Success Rate and Time: The overall recall rate in the study was 92.1% (95% confidence interval: 91.05%-93.13%). From the 2594 participants, 205 failed to enter their pass-gestures within five attempts. We divided participants into two groups depending on their success in the recall task and present their data in Table II. Once again, due to the the simpler format of our gestures, recall times are quick (5.11 seconds) compared to those reported in prior studies of gesture passwords (16.49 seconds) [10]. However, recall rates are low - for short-term recall, figures of 98.9% have been previously reported [10]. Examining the data in detail, its clear that the attack gesture task strongly impacted recall rates - for participants who recalled their pass-gestures, 68% of errors involved entering a match for their attack-gesture; for participants who failed recall this figure was 60%. We also note that, from the set of 2389 participants who correctly recalled their gesture, the number of attempts in which an entered gesture failed to match either pass- or attack-gestures was 0.11 - this figure corresponds closely to the number of errors (12.1%) recorded in an in-the-wild study of Android pattern locks [27]. We argue this suggests that the somewhat reduced recall rates recorded in this study are due to the attack gesture distracter task rather than a reflection of fundamental user performance during gesture authentication recall.

F. Security Results: Preprocessing and EER

To evaluate the security of the collected gesture passwords, we apply methods from the initial stage of our framework (see

TABLE I
Setup Cancels, Failures and Times in the First Study (μ : mean,
σ : standard deviation, $\tilde{\mu}$: median)

	Pa	ass-Gestu	re	Att	tack-Gesti	ure
Measure (units)	$\mid \mu$	σ	$\tilde{\mu}$	μ	σ	$\tilde{\mu}$
Setup Cancels (#)	0.6	2.58	0	0.14	0.64	0
Confirm Failures (#)	0.14	0.52	0	0.77	1.88	0
Setup Time (s)	24.38	30.13	15.52	17.82	22.08	11.48

Section III-C). We first studied and described the gestures. To gain an understanding of the types of gesture produced, we adapted an existing categorization of gesture passwords from prior work featuring seven categories: digits, geometric shapes, letters, math functions, math symbols, music symbols, and special characters [11]. Two independent raters manually examined the same random 10% of the gestures, categorizing each according to this prior scheme and extending it where necessary. They then discussed their ratings, agreed on four additional categories - compound, cursive, iconic and other - and applied this new scheme to a second random 10% of the gestures. More details on these categories and the results from this process are shown in Appendix A. The results are broadly similar to those in a prior study [11], suggesting that the gestures captured in this work are reasonable. In addition, Appendix B shows the distribution of all raw points input in creation gestures, binned into three by three grids with start and final points separated out. The data suggests that, as with other forms of graphical password such as patterns [14], users tend to start strokes from the top left of the available drawing area and, to a lesser extent, finish them in the bottom right. In contrast, points entered during the course of an ongoing gesture were more central - less prevalent around all edges (and particularly corners) of the drawing area.

We measured FRRs from each user's creation, confirmation and, if present, recall gestures. We generated FAR data from the full set of all users' creation gestures. For Protractor, EERs improved consistently with increasing re-sampling sizes, so we opted to set the re-sampling size to 96 (the default for Protractor's reference implementation). For DTW, minimum EERs were observed when gestures were re-sampled to 24 points, slightly larger than the 16 point optimal value reported in prior work [11]. At these values, the EER for DTW is 3.59% (AUROC 0.984 and corresponding to a DTW distance of 18.4) and for Protractor 4.14% (AUROC 0.974 and corresponding to an inverse cosine distance of 1.25). Figure 3, shows the ROC curves contrasting these algorithms. We note the EER value for Protractor is somewhat lower than reported in the literature - Sherman et al. [3], for example, report 7.07% and 15.97% with Protractor for two different data sets and when matching against two templates. Examining their data in detail suggests the reduction we observe is due to a lower FRR, rather than any change in FAR. This suggests the difference is predominantly due to our decision to conduct the study with a permissive match threshold (1.0) to gather a greater range of user inputs. While no specific match threshold is reported

TABLE II

Recall failures due to matching attack gestures (Attack entries) or no gesture (No match entries) and recall times in the First Study (μ : mean, σ : standard deviation, $\tilde{\mu}$: median)

	Successful Recall			Unsuc	cessful	Recall
Measure (units)	μ	σ	$ ilde{\mu}$	μ	σ	$ ilde{\mu}$
Attack entries (#)	0.23	0.66	0	3	1.81	3
No match entries (#)	0.11	0.39	0	2	1.81	2
Recall Time (s)	5.11	5.52	3.12	N/A	N/A	N/A



Fig. 3. Receiver Operating Characteristic (ROC) curves contrasting relative FRR and FAR performance for Protractor and DTW recognizers. The Protactor EER is 4.14% (AUROC: 0.974) at a threshold value of 1.25 and the DTW EER is 3.59% (AUROC: 0.984) at a threshold value of 18.4.

in [3], if we assume use of a stricter value, such as the 2.0 used in related work [10], a greater number of false rejections, and correspondingly, a higher EER would be expected.

G. Security Results: Entropy Analysis

We then followed the entropy analysis processes in our framework. We created 270 different 2-gram models and applied selection criteria to choose a subset of models for optimization, as described in Section III-D. We used DTW to calculate these metrics due to this algorithm's improved performance over Protractor in terms of EERs, and set the threshold t to the value corresponding to 10% FRR, one of the standardized FRR levels introduced in our assessment framework (see Section III-E). Appendix C includes results in terms of our selection criteria for all models achieving a crack rate of greater than 10%. We choose three models for optimization - see Table III. The first achieves the overall best crack rate and is based on discretization into two length and ten angular regions ("2x10"). However, gestures reconstituted from this model show a relatively low similarity to the original user-chosen gestures, suggesting some information in complex strokes may be lost. Accordingly, we selected two additional models with increasing numbers of either length ("3x10") or both length and angle ("4x12") discretization regions that combine high crack rates with improvements in similarity and small reductions in model completeness. The outcomes from the optimization process are also depicted in Table III. Appendix D shows the optimized boundary region sizes and distribution of start, center and final strokes in these three models. These figures show that initial strokes tend to involve vertical and/or right movement (matching the gesture start locations shown in Appendix B), while central strokes are short (likely due to users drawing curves) and final strokes have a relatively even distribution. Based on a review of this material, we believe that the model based on discretization into three length and ten angle regions (3x10) provides a well balanced combination of high crack rate, close accuracy to the original user-chosen gestures, and high proportion of

TABLE III SELECTED *n*-gram Models after optimization showing Crack Rate (CR), Similarity (SM) and Completeness (CP) metrics.

N/			Model P	Parameters		Mod	lel Perform	ance
ivame	Len	Ang	Phase	Smoothing	Excl.	CR	SM	CP
2x10	2	10	offset	Good-Turing	single	18.24%	62.72%	94.78%
3x10	3	10	offset	Add-1	dual	16.85%	86.16%	90.73%
4x12	4	12	offset	Add-1	dual	15.46%	84.70%	73.67%

TABLE IV Comparison of partial guessing entropy ("bits of information") with cracking fraction (α) across password data sets.

Dataset				α		
Duiusei	0.1	0.2	0.3	0.4	0.7	1.0
2x10 Pass-gestures	6.29	8.39	11.39	13.31	16.11	17.98
3x10 Pass-gestures	6.97	9.69	13.26	15.41	18.57	20.68
4x12 Pass-gestures	7.47	11.27	15.94	18.40	21.68	23.98
4-digit PINs [2]	5.19	7.04	8.37	9.38	11.08	11.83
Patterns [14]	5.04	5.82	6.54	7.19	9.20	12.71

observed *n*-gram cases. We note that the model based on discretization into two length and ten angle regions (2x10) may perform better in terms of crack rate when gestures in a set are relatively simple.

Partial guessing entropy results from the three optimized models, converted to "bits of information" are shown in Table IV. The results of two additional data sets are included: 4-digit PINs [2], and screen lock patterns [14]. The results indicate that gestures, across all three n-gram models, potentially have higher entropy estimates compared to PINs and patterns at different α levels. Although our *n*-gram models can be further optimized in the future (to more accurately estimate the probabilities of real-world gesture passwords), these early comparisons provide some evidence that guessing gestures might be more challenging compared to PINs or patterns. We note the models show a steep rise in partial guessing entropy levels between α values of 0.1 and 0.4. This likely reflects the "weak subspace" of gestures [11] – the idea that a subset of user created gestures (at low α levels) take simple forms that are relatively easy to guess, while the remainder (encountered at higher α levels) are more challenging.

H. Security Results: Clustering-based Dictionary Attack

We then applied methods from the third stage of our framework: performing clustering-based dictionary attacks on the collected gestures and measuring cracked rates. Following the processes outlined in Section III-E, we generated clusters, dictionaries and attack data for DTW and Protractor recognizers. We report data for the full gesture sets. Dictionaries are shown in Figure 4. Protractor led to a total of 325 clusters with a mean (inverse) inter-cluster distance of 4.69 for the top 20 dictionary clusters. DTW resulted in a total of 290 clusters and a mean inter-cluster distance of 8.97 for the top 20 dictionary clusters. These thresholds are substantially more permissive than EERs thresholds, indicating that, in general, gestures in each cluster would be matched with each other if EER thresholds are used as a criteria. This suggests the clustering algorithm was effective at grouping similar gestures.



Protractor dictionary

DTW dictionary

Fig. 4. Dictionaries created from clustering the full sets of pass-gestures in the first study for both Protractor (left) and DTW (right).



Fig. 5. Proportion of cracked gestures with dictionaries at FRRs from 0%-15% with Protractor and DTW recognizers in the first study. Vertical lines show EER values for each recognizer.

As we are primarily interested in online attack on mobile phone unlock screens, we used dictionaries generated from training sets to guess gesture passwords when k = 20, the default number of guesses before an Android device locks out further attempts. The results, for both dictionaries and recognizers, are shown in Figure 5. At thresholds corresponding to EER values, the dictionaries are highly effective. They crack between 54.18% (DTW) and 58.37% (Protractor) of gestures. In general, fewer gestures are guessed with DTW than Protractor, suggesting that future gesture authentication systems should use DTW in preference. More generally, this data also highlights that data-set wide EERs may overestimate the security of gesture passwords - stricter thresholds need be applied to create a viable system. The limited data on thresholds used in prior work confirms this – a threshold of 2.0 with Protractor has been previously proposed [10]. In our data set, increasing the Protractor threshold to 2.0 corresponds to an FRR for genuine users of 11.54%, which would reduce the effectiveness of the clustering-based dictionary attack against Protractor from 58.37% to 31.49%. Applying a threshold corresponding to the same 11.54% FRR to DTW drops the performance of clustering attack from 54.18% to 23.13%. For comparison, dictionary attacks performed on patterns achieve between 13.33% crack rate in a real-world mobile application [28], and 32.55% in an MTurk study [14] and realworld pattern lock error rates, FRRs, are 12.1% [27].

We note our dictionary attacks are more effective than the offline guessing attack reported in Liu *et al.* [11] – they crack a comparable 55.9% of gestures using DTW but required 10^9 guesses. There are a number of reasons for this: the gestures generated in more controlled settings may be more complex, distinct and consistent [29] than those generated by MTurk workers online (though which group generates content more representative of genuine password gestures is an open question), and the gestures used in the current work are intentionally constrained to be simple single strokes in the small,

practical "screen-lock" region of smartphones rather than the multi-stroke full-screen gestures that have been previously studied [5]. However, the strength of the dictionaries likely reflects the presence of a "weak subspace" [11] of gesture passwords – a large subset of the gestures users create are insecure due to, for example, their similarity to common reference points such as letters, their simplicity or the tendency for other users to create highly similar strokes. We identify the lack of diversity in users' gestures revealed by this analysis as a major problem undermining the potential of gesture passwords as a smartphone lock scheme.

V. SECOND STUDY: GESTURE SELECTION POLICIES

To increase the entropy of gesture passwords, we conducted a second study exploring the impact of four policy conditions: a standard *baseline* condition, similar to that used in the first study; a novel lexical policy in which participants were provided with words that could inspire their gestures; a novel blacklist policy that prevents users from creating pass-gestures matching those in a displayed dictionary and; a consolidated blacklist policy with a refined set of blocked gestures. The lexical, blacklist and consolidated policies were intended to increase the entropy of the gestures participants generate by, respectively, inspiring users to create more diverse forms and restricting the use of dictionary items. We apply our security assessment framework to data from this study to determine if resistance to dictionary based attacks is improved; this work also serves to further validate our framework. We also report on usability outcomes from the policies over two recall study sessions. The ethical aspects of the study were IRB approved.

A. Blacklist Policy Design

For the *blacklist* policy we created a dictionary consisting of the representative gestures from the largest 20 clusters of pass-gestures in the first study. We presented them to participants and informed them they could not use these gestures as their pass-gesture. We enforced this by testing creation pass-gestures against the dictionary and generating a policy violation error message if there was a match; this policy explicitly checked for compliance. The dictionary is only shown during the pass-gesture creation phase.

B. Consolidated Policy Design

The *consolidated* policy design used the full set of clusters from the first study. We ordered the clusters by size and extracted the central representative gestures. We then traversed the ordered list to create a consolidated set of clusters by matching each subsequent gesture against the set of those already examined (using Protractor and the first study EER value of 1.25). In the case of a match, the list cluster was merged with the one in the consolidated set. If there was no match, the list cluster was added to the set. After examining all original clusters, we produced a consolidated dictionary from the 20 largest clusters in the consolidated set. This process aimed to produce a dictionary with minimal replication by merging small clusters of similar gestures to create a more representative set of final clusters and gestures. Beyond these differences in dictionary generation, the consolidated policy was identical to the blacklist policy in how it was presented.

C. Lexical Policy Design

The lexical policy displayed words, which users can update, during pass-gesture creation and indicated participants can use these terms to inspire their gestures. The words are shown only during the initial gesture creation and not stored with the gesture templates or intended to serve as mnemonic cues or "gesture hints." The policy resembles prior gesture password policies [21] in some ways: it is not enforceable and seeks to provide open-ended guidance for creating diverse or unique gestures. It differs by presenting semantic content rather than guidance or advice. For this policy we constructed a dictionary of sixty English words starting from a combined set of 3000 monosyllabic [30] and 3000 disyllabic [31] words rated for imageability, a construct which captures how easily a given word elicits or evokes a mental image or picture of what it refers to. While this clearly relates to the idea that words may inspire gestures, the relationship is hard to predict. High imageability words might lead to a large number of similar gestures (e.g., all gestures for "ice-cream" might be frosty cones), while low imageability words might have more diversity, as users think of different things, or might poorly support the task by failing to inspire gestures at all. Consequently, our dictionary was built from a spread of words: 20 words from the top end of the imageability scale, 20 words from the center of the scale and 20 words from the bottom of the scale. We also filtered words by further criteria to ensure they avoided extremes of affect [35] and were widely known [34], [33]; see Table V for full details.

D. User Study Design

This study was largely similar to first study; it used MTurk, ran on a website, required completion on a mobile device and used the same Protractor gesture recognizer. In addition to the introduction of the four policy conditions, there were a number of differences: Protractor was adjusted to reflect the first study EERs with an elevated match threshold of 1.25 – we kept this low threshold to maintain our ability to capture the broadest range of possible gestures; we introduced a gesture practice phase, common in similar studies [36]; we replaced the attack gesture task, due to its impact on recall rates, with a typical

TABLE V
WORDS SETS USED IN LEXICAL POLICY – REPRESENTATIVE EXAMPLES
AND DESCRIPTIVE STATISTICS (μ : MEAN, σ : STANDARD DEVIATION)

Imageability Level	Hi	gh	Med	lium	Low		
Example words	Rug,	Boat	Cruise	e, Plot	Echo,	Echo, Union	
Measure (units)	$ \mu $	σ	μ	σ	μ	σ	
Imageability Rating (0-7) [30], [31]	6.59	0.23	4.79	0.99	3.83	0.72	
Concreteness Rating (0-5) [32]	4.9	0.13	3.72	0.73	2.83	0.66	
Prevalence (z-score) [33]	2.4	0.1	2.42	0.08	2.44	0.12	
Age of acquisition (years) [34]	4.26	0.59	7.42	1.61	7.75	1.23	
Valance Rating (0-7) [35]	5.66	0.55	5.6	0.56	5.54	0.49	
Arousal Rating (0-7) [35]	3.78	0.45	3.76	0.53	3.73	0.51	
Dominance Rating (0-7) [35]	5.72	0.46	5.67	0.38	5.64	0.5	



Fig. 6. The second study on a Samsung Galaxy 8 Android mobile phone in Google Chrome. Blacklist and Consolidated policies presenting banned dictionary gestures (a), (b). Lexical policy with the term "shadow" displayed to inspire gestures; it can be changed via the bottom left button (c). Both the gesture dictionary and lexical word disappear as soon as a user's finger lifts from the screen during pass-gesture creation; neither is shown again.

memory game distracter task and; there was a follow-up recall session 24-72 hours after the initial session – a recall period similar to that used in prior work [14] to reflect real-world smart phone unlocking frequencies. For this more involved study, participants received 1 USD for completion of the first session and 0.5 USD for completion of the second session. Details of the study structure are described below; for brevity, we omit details that replicate those in the first study.

Study instructions were similar to the first study - they were modified to reflect the revised study structure and included a brief description of the policy; each participant completed only one policy condition. Although participants did not create attack-gestures, we retained this section of the instructions to motivate participants to create stronger gesture passwords. In the baseline policy condition, creation and confirmation of the pass-gesture proceeded identically to the first study. In the blacklist and consolidated policy conditions, participants were shown the gesture dictionary on the password creation screen and informed they cannot use these gestures; all creation passgestures were tested against the dictionary. If there was a match, participants received a policy error message and needed to restart the pass-gesture creation process. In the lexical policy condition, the creation screen prominently displayed a word, the instruction to use this word to inspire their gesture, and the ability to change this word to another one. The first presented word was determined by the participant ID number (from the database logging the study results) in order to more evenly balance initial word presentation. Changed words were delivered in a random order. Participants were able to change the word multiple times. Figure 6 shows all policy interfaces.

After confirming their pass-gestures, participants practiced them 10 times. Gestures during practice were matched against creation and confirmation gestures and feedback as to whether they matched was provided to users after each gesture entry. Success was not mandated. Following prior study designs [14], we then had participants complete a distracter task to wipe their short term memory: they played a color matching game in which they flipped over pairs of cards looking for similar colors. The study session finished with a recall task and closing feedback that was identical to the first study. Those participants who successfully recalled their gestures within five attempts were invited back to a second recall session. This was achieved via the bonus feature of MTurk that allows the owners of a task to send an email and provide an additional payment to workers who complete their task. The email was sent 24 hours after completion of the study and included a link and a QR code for the recall study. The follow up task was a single recall test, identical in structure to the initial recall test. The link expired after 48 hours of receiving the invite.

E. Measures

We measured all data captured in the first study. In addition, we logged the number of practice gestures participants correctly matched against their creation or confirmation templates and a new set of recall data for participants who completed the second session. For participants in the blacklist and consolidated policies we additionally logged the number of times they matched the dictionary gestures during pass-gesture creation while in the lexical policy we logged the lexical cue displayed when they created their gesture, and the number of times they changed this cue.

F. Usability Results

1) Demographics: In total, 4000 unique US based MTurk workers completed the study: 1000/policy. They were predominantly white (65.1%), black or African American (11.4%), Hispanic (9.7%) or Asian (7.8%) and 18-24 (23.5%), 25-34 (48.6%) or 35-44 (19.5%) years old. Education level (post-graduate (11.7%), college (49.2%), high school level (37%)) and profession were diverse; 11% were students. Participants completed the study tasks on the first day (from the start of gesture creation until the final gesture recall) in a median of 132 seconds, and the gesture password recall task (equivalent to the recall time measure, see section IV-C) on the second day in a median of 4 seconds; the hourly compensation rates were 27 USD per hour and 450 USD per hour respectively.

2) Setup, Participation and Recall: Usability measures are summarized in Table VI while participation and recall rates are in Table VII. For the majority of measures, the data was homogeneous and exhibited strong positive skews; accordingly we applied non-parametric testing. Kruskal-Wallis tests on Setup Time ($\chi^2 = 1076.5, p < 0.001$), Day 1 Recall Time ($\chi^2 = 133.15$, p < 0.001) and Day 2 Recall Time $(\chi^2 = 34.525, p < 0.001)$ revealed significant differences. Bonferroni corrected post-hoc Wilcoxon tests on these measures revealed the following comparisons to be significant: all differences in Setup Time (at p < 0.001); all bar between baseline and lexical in Day 1 Recall Time (at p < 0.003) and; between baseline-blacklist, baseline-consolidated and lexicalconsolidated in Day 2 Recall Time (at p < 0.003). Recall Attempts did not vary on day 1 ($\chi^2(2) = 5.66, p = 0.129$) or day 2 ($\chi^2(2) = 4.43$, p = 0.22). Bonferroni corrected Chi-squared tests of independence revealed the following differences in participation and recall rates: the consolidated condition recorded reduced Day 1 Recall Rate compared to



Fig. 7. Receiver Operating Characteristic (ROC) curves contrasting FRR and FAR performance for the four policy conditions with Protractor and DTW.

baseline ($\chi^2 = 9.39$, p = 0.013); the blacklist condition showed reduced Day 2 Return Rates compared to consolidated ($\chi^2 = 7.5$, p = 0.037) and; blacklist and consolidated led to lower Day 2 Recall Rates than baseline (respectively, $\chi^2 = 7.04$, p = 0.048 and $\chi^2 = 7.46$, p = 0.038). In sum: our policies lengthened mean setup (by 9.63 to 53.21 seconds) and recall (by 0.21 to 1.1 seconds) times and led to modest reductions (of approximately 3%) in recall rates.

3) Policy Specific Measures: In the lexical policy, only 24.2% of participants opted to change the word. They did so a median of 3 times. This, in conjunction with the setup times close to baseline, suggests a low degree of compliance with the policy: most participants did not engage strongly with the task. In the blacklist and consolidated policies, participants attempted to create a gesture that matched one in the dictionary a mean of, respectively, 1.99 times (σ : 3.24) and 2.44 times (σ : 2.96). While this shows participants did experience additional challenges in creating gestures due to the blacklisted dictionaries, its not sufficient to account for the substantial increase in setup times alone – rather than make repeated attempts, the blacklist and consolidated conditions led to participants engaging more strongly with the task; spending more time thinking about the gesture they would create.

G. Security Results: Preprocessing and EER

Appendix A shows a subjective categorization of gestures from each policy, using the same categories and raters from the first study. We note that gestures in the blacklist and consolidated conditions show increased rates of compound gestures (complex gestures involving two or more distinct forms) compared to baseline: both 39.0% versus baseline's 16.6%. Additionally, lexical shows the highest rates for the simple categories of geometric shapes and letters (54% in total). Appendix B shows the distribution of all raw points in creation gestures across all policies. We calculated EERs as in the first study; optimal re-sampling sizes were unchanged at 96 for Protractor and 24 for DTW. Table VIII shows the EERs and the corresponding threshold values and Figure 7 the ROC curves. In general, the lexical condition shows poor performance: EERs are elevated compared to baseline. The blacklist and consolidated conditions show more promise: EER rates are similar (Protractor) or improved (DTW) over baseline. At the same time, thresholds become more permissive. These trends suggest that the blacklist and consolidated policies collected a more diverse set of gestures – EER thresholds are more permissive due to greater variance in the strokes generated.

H. Security Results: Entropy Analysis

As the second analysis, we generated *n*-gram models for each policy using the well balanced 3x10 model configuration from the first study. Appendix E shows the stroke distribution for each policy in Figure 11. We then calculated partial guessing entropy. The results are shown in Table IX. All three of the policies introduced in this paper equal or exceed the baseline policy at most α levels. However, there is some discrepancy between the policies as α increases. Specifically, at the low α levels more relevant to an online attack with a small number of guesses, the blacklist and consolidated policies show elevated partial guessing entropy. In contrast, the lexical policy shows a reduction in partial guessing entropy compared to baseline at low α levels while, at higher levels of α , it achieves peak results. We suggest these variations may be due to compliance rates with the lexical policy: the minority of users who engaged more strongly with the lexical cue may have created highly unique and complex gestures, making it more difficult for an attacker to guess the full set. In contrast, the enforced compliance with the blacklist and consolidated policies ensured that all users needed to create gestures reflecting the policy constraints, resulting in a reduction in the size of the "weak subspace" of easily guessable gestures occupying low α levels.

I. Security Results: Clustering-based Dictionary Attack

Table VIII shows the number of clusters and the mean intercluster distances for each algorithm and policy. We note that the number of clusters are reduced compared to the first study due to the smaller sample sizes (1000 vs 2594); this likely also accounts for some of the variations in mean inter-cluster distances between the two studies. Looking at the differences between the policies, we note the similarly between gestures in each dictionary cluster is markedly higher in the lexical policy than in baseline. This suggests that gestures in each cluster in the lexical policy were more homogeneous than in baseline. In contrast, the blacklist policy (for Protractor) and the consolidated policy (for both recognizers) showed more heterogeneity in the gestures within each cluster. The clusters in these policies collected a more diverse set of gestures. The results of dictionary attacks on all four policies are shown in Figure 8. The charts suggest the lexical policy impairs resistance to the dictionary attacks, while the blacklist and consolidated policies may enhance it. We tested these variations for significance at specific FRR values of 2.5% (DTW-only), 5% and 10% using sets of Bonferroni corrected Chi-squared tests of independence. For Protractor, all differences in the

TABLE VI

USABILITY RESULTS FROM THE SECOND STUDY (μ : MEAN, σ : STANDARD DEVIATION, $\tilde{\mu}$: MEDIAN). TABLE SHOWS # SETUP CANCELS (SC), # CONFIRM FAILURES (CF), SETUP TIME (S) (ST), # PRACTICE MATCHES (PM), DAY 1 RECALL TIME (S) (D1-RT), # DAY 1 RECALL ATTEMPTS (D1-RA), DAY 2 RECALL TIME (S) (D1-RT) AND # DAY 2 RECALL ATTEMPTS (D1-RA) FOR ALL POLICIES.

		Baseline			Lexical			Blacklist		(Consolidate	d
Measure	μ	σ	$\tilde{\mu}$	μ	σ	$ ilde{\mu}$	μ	σ	$ ilde{\mu}$	μ	σ	$ ilde{\mu}$
SC	0.78	2.55	0.0	0.81	1.76	0.0	0.31	1.22	0.0	0.32	1.22	0.0
CF	0.2	0.62	0.0	0.25	0.64	0.0	0.27	0.61	0.0	0.35	0.7	0.0
ST	29.48	41.21	17.26	39.11	37.83	27.87	67.35	69.57	48.96	82.69	94.01	59.64
PM	9.33	1.69	10.0	9.14	1.94	10.0	8.99	2.14	10.0	8.94	2.17	10.0
D1-RT	3.03	3.25	2.13	3.24	3.53	2.29	3.54	4.03	2.58	3.92	3.85	2.8
D1-RA	1.08	0.4	1.0	1.12	0.52	1.0	1.07	0.36	1.0	1.09	0.41	1.0
D2-RT	5.13	5.4	3.55	5.61	5.88	3.7	6.86	11.63	4.09	6.23	6.01	4.23
D2-RA	1.22	0.61	1.0	1.25	0.63	1.0	1.27	0.62	1.0	1.29	0.71	1.0

performance of clustering dictionary attack were significant (at $p \leq 0.0014$ or lower, $\chi^2 = 13.6$ to 149.32) except between blacklist and consolidated (all $p \geq 0.27$, $\chi^2 = 0.78$ to 4.02) and between baseline and consolidated at the 5% FRR level (p = 0.2, $\chi^2 = 4.57$). Data for DTW tells a similar story. All differences were significant except between blacklist and consolidated (all p = 1). Blacklist significantly improved over baseline at the 5% FRR level (p = 0.029, $\chi^2 = 7.95$) and all other significant differences were at p < 0.001 ($\chi^2 = 17.48$ to 609.9). To provide some specifics: using DTW and at the 11.54% FRR level derived from prior work and discussed in the first study, the best performing policy is consolidated. The dictionary attack cracks 14.93% of gestures generated under the consolidated policy, an improvement of 30.69% over the 21.54% cracked in the baseline condition at the same FRR.

We conclude our blacklist and consolidated policies improve the security of gesture passwords, while our lexical policy reduces it. A possible explanation is that the lexical policy *does not require* compliance, while it is mandated in other two policies, ensuring that participants can not use one of the blocked gestures. In addition, participants generated relatively simple (see Appendix A) and homogeneous (see Table VIII) gestures with the lexical policy. We also note the lack of significant security improvements with the consolidated policy compared to the blacklist policy suggests that this more comprehensive set of restrictions on gestures may only provide marginal benefits. Finally, we also note the results are aligned with those from the first study: DTW outperforms Protractor by offering an increased resistance to dictionary attacks.

TABLE VII
Participation and recall rates in the second study; 500
PARTICIPANTS STARTED DAY 1 IN EACH POLICY. ALL PROPORTIONS
SHOWN NUMERICALLY, AS % AND WITH CONFIDENCE INTERVALS (CI).

	Day 1	Da	y 2
	Recall Rate	Participants	Recall Rate
	(#, %, CI)	(#, %, CI)	(#, %, CI)
Baseline	1000,983, 98.3%,	570, 57.98%,	559, 98.07%,
Daschille	97.49%-99.1%	54.9%-61.07%	96.94%-99.19%
Levicel	1000,977, 97.7%,	546, 55.88%,	526, 96.33%,
Lexical	96.77%-98.62%	52.77%-58.99%	94.76%-97.91%
Placklist	1000,969, 96.9%,	516, 53.25%,	490, 94.96%,
DIACKIISU	95.82%-97.97%	50.1%-56.39%	93.07%-96.84%
Concolidated	1000,959, 95.9%,	571, 59.54%,	542, 94.92%,
Consolituateu	94.67%-97.12%	56.43%-62.64%	93.12%-96.72%



Fig. 8. Crack rates in each policy using Protractor and DTW dictionaries.

VI. DISCUSSION

By applying the proposed framework to the first study, we made two integral observations: (1) EERs alone cannot adequately represent the security of gesture passwords, often overestimating their security, and (2) our clustering-based dictionary attack is highly effective, and many baseline gesture passwords are vulnerable to online attacks due to the biased ways in which people select easy-to-draw (simple) gestures.

Password policies may be able to mitigate these biases. However, designing policies is challenging and characterized by trade-offs; increasing security reduces usability [37]. Our three gesture policy designs sought to improve security while minimizing costs to usability through two different strategies: the lexical policy sought to inspire and the blacklist and consolidated policies to restrict. The lexical policy maintained usability, but reduced security. A major problem was compliance; behavioral data (in terms of setup time and word update rate) suggest participants did not engage with the task. Alternatively, the inherent imageability [30] of words may have led them to create homogeneous forms - this assertion is backed up by the high similarity of gestures within each cluster (see Table VIII) and data from the qualitative categorization of the gestures that indicates the lexical policy led to a high proportion of gestures based on simple shapes and letters - see Appendix A. On the other hand, the blacklist and consolidated policies enforced compliance and improved security at the cost of increased setup and recall times. This may be due to increased heterogeneity of the gestures: in qualitative categorization, they led to high rates of compound categorizations, referring to gestures with two or more elements, that we suggest will

TABLE VIII

POLICY COMPARISON DATA: EER THRESHOLDS, PERCENTAGES, AUROC VALUES, MEAN INTER-CLUSTER DISTANCES FOR LARGEST 20 CLUSTERS AND NUMBER OF CLUSTERS. NOTE: LARGER PROTRACTOR DISTANCES AND SMALLER DTW DISTANCES REPRESENT MORE SIMILAR GESTURES.

			Protracto	r				DTW		
Dataset		EER		Inter-Cluster	Number		EER		Inter-Cluster	Number
	Threshold	%	AUROC	Match Scores	Clusters	Threshold	%	AUROC	Match Scores	Clusters
Baseline	1.32	3.19%	0.974	3.01	140	16.24	2.28%	0.989	10.2	121
Lexical	1.39	4.82%	0.963	4.44	125	17.91	4.3%	0.982	8.66	119
Blacklist	1.14	3.34%	0.968	2.61	141	17.31	1.68%	0.993	9.4	128
Consolidated	1.1	3.35%	0.969	1.55	131	18.16	1.83%	0.993	13.05	118

TABLE IX Comparison of partial guessing entropy ("bits of information") with cracking fraction (α) across policy condition datasets.

Dataset				α		
Dulusei	0.1	0.2	0.3	0.4	0.7	1.0
Baseline 3x10	7.55	11.06	14.47	16.23	19.12	21.08
Lexical 3x10	7.53	11.59	15.27	16.99	19.64	21.37
Blacklist 3x10	8.29	12.99	15.62	16.99	19.43	21.2
Consolidated 3x10	8.42	13.31	15.75	17.06	19.45	21.2

be more diverse (and thus harder to guess) than simpler single forms. Blacklist policies should be refined in future work.

It is also worth comparing the gestures in this paper with pattern locks, which can be considered as constrained gestures - Cho et al. [14] provide a detailed analysis that includes cracking results against patterns captured in a similar setting to this work (on MTurk) and in a standard policy: in 20 guesses, 33% of patterns were cracked. With our baseline policy, achieving a similar level of security would require a more rigid match criteria than used in our studies. A threshold of 2.0, used in prior work [10], would achieve this (23.13% to 31.49%) cracked) at a cost of increased FRRs - based on data in the first study, these would rise to 11.54%. We note this figure remains reasonable; pattern lock errors are reported to run at 12.1% in real world use [27]. While our blacklist policies offer an improvement on performance in terms of resistance to dictionary attack (up to 14.93%), they do so at a cost in other metrics. Cho et al. [14] report pattern setup times of 22 seconds, whereas our blacklist policy and consolidated polices took between three and four times as long, reflecting participants' mandated engagement with the gesture password creation task. While this is not ideal, free-form gesture passwords are novel and setup times may improve with more training and use in the future. There are further limitations to this work. MTurk workers, and the gesture passwords they create, may not be representative [29], and the security and usability incentives we offered upon gesture selection may not match the realworld intentions and needs. Attackers may be able to use other sources of data (e.g., smudges [28]) to aid guessing attacks. Alternative discretization methods may improve guessability in *n*-gram models, and affect the partial guessing entropy results. Larger data sets collected in the wild (e.g., through a realworld mobile application) are a precursor to addressing these challenges - we identify exploring alternative ways to gather real-world samples as a challenge for future work.

It is also worth reflecting our characterization of gesture passwords: we position them against the dominant (primary) unlock methods of PINs and patterns. In this scenario, gesture passwords improve on partial guessing entropy. Alternatively, they could be positioned against secondary unlock methods (e.g., biometrics) used in addition to a primary scheme. Gesture passwords may still provide value in this case as a complementary usable and secure secondary authentication method. We note current smart-phones provide multiple secondary unlock options (e.g., face recognition, iris scanner, and fingerprint scanner) and allow users to operate different schemes interchangeably. Indeed, multiple options are needed due to limitations affecting the use of each secondary scheme: fingerprint scanners perform poorly when fingers are oily or dirty and recognition rates may decrease with user age [38]; face recognition and iris scanning do not work in poor lighting, or when users are mobile or otherwise not looking at the phone screen. Accordingly, users set up multiple authentication systems to unlock their devices in various situations. As such, we believe gesture passwords, which can be readily entered without close attention to the phone screen (i.e., they are scale and position independent), provide a valuable alternative means for users to quickly and easily unlock their phones. We believe they would be beneficial in common situations such as when users are not attending to their phone screen (e.g., while walking or performing other activities [38]).

VII. CONCLUSION

This paper proposes a novel security assessment framework for gesture passwords based on entropy assessment and automated clustering and dictionary generation capabilities. It validates this framework by collecting and analyzing the largest current sample of gesture passwords (N=2594). Responding to the ease with which gesture passwords were cracked, we propose three novel policy designs for gesture passwords: blacklist, consolidated and lexical. These are evaluated in a new two-day study (N=4000). While the lexical policy reduces security with respect to a online attack with 20 guesses, the blacklist and consolidated policies boost guessing entropy and reduce susceptibility to dictionary attacks at a modest cost in terms of usability. We believe gesture passwords are a promising smartphone authentication technology and that future work should focus on understanding the gestures people create and the policies, guidance, and feedback they need to do so more securely. This paper works towards this goal.

ACKNOWLEDGMENT

This work was supported by Samsung Research and the NRF of Korea (NRF-2019R1C1C1007118).

REFERENCES

- [1] S. Uellenbeck, M. Dürmuth, C. Wolf, and T. Holz, "Quantifying the security of graphical passwords: The case of android unlock patterns," in *Proceedings of the 2013 ACM SIGSAC Conference on Computer & Communications Security*, ser. CCS '13. New York, NY, USA: ACM, 2013, pp. 161–172. [Online]. Available: http://doi.acm.org/10.1145/2508859.2516700
- [2] H. Kim and J. H. Huh, "PIN selection policies: Are they really effective?" Computers & Security, vol. 31, no. 4, 2012.
- [3] M. Sherman, G. Clark, Y. Yang, S. Sugrim, A. Modig, J. Lindqvist, A. Oulasvirta, and T. Roos, "User-generated free-form gestures for authentication: Security and memorability," in *Proceedings of the 12th Annual International Conference on Mobile Systems, Applications, and Services*, ser. MobiSys '14. New York, NY, USA: ACM, 2014, pp. 176– 189. [Online]. Available: http://doi.acm.org/10.1145/2594368.2594375
- [4] A. De Luca, E. von Zezschwitz, N. D. H. Nguyen, M.-E. Maurer, E. Rubegni, M. P. Scipioni, and M. Langheinrich, "Back-of-device authentication on smartphones," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '13. New York, NY, USA: ACM, 2013, pp. 2389–2398. [Online]. Available: http://doi.acm.org/10.1145/2470654.2481330
- [5] C. Liu, G. D. Clark, and J. Lindqvist, "Where usability and security go hand-in-hand: Robust gesture-based authentication for mobile systems," in *Proceedings of the 2017 CHI Conference* on Human Factors in Computing Systems, ser. CHI '17. New York, NY, USA: ACM, 2017, pp. 374–386. [Online]. Available: http://doi.acm.org/10.1145/3025453.3025879
- [6] A. Pirhonen, S. Brewster, and C. Holguin, "Gestural and audio metaphors as a means of control for mobile devices," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '02. New York, NY, USA: ACM, 2002, pp. 291–298. [Online]. Available: http://doi.acm.org/10.1145/503376.503428
- [7] T. Nguyen and N. Memon, "Tap-based user authentication for smartwatches," *Computers & Security*, vol. 78, pp. 174 – 186, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/ pii/S0167404818303778
- [8] A. Oulasvirta, S. Tamminen, V. Roto, and J. Kuorelahti, "Interaction in 4-second bursts: The fragmented nature of attentional resources in mobile hci," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '05. New York, NY, USA: ACM, 2005, pp. 919–928. [Online]. Available: http://doi.acm.org/10.1145/1054972.1055101
- [9] N. Sae-Bae, K. Ahmed, K. Isbister, and N. Memon, "Biometric-rich gestures: A novel approach to authentication on multi-touch devices," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '12. New York, NY, USA: ACM, 2012, pp. 977–986. [Online]. Available: http://doi.acm.org/10.1145/2207676.2208543
- [10] Y. Yang, G. D. Clark, J. Lindqvist, and A. Oulasvirta, "Free-form gesture authentication in the wild," in *Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems*, ser. CHI '16. New York, NY, USA: ACM, 2016, pp. 3722–3735. [Online]. Available: http://doi.acm.org/10.1145/2858036.2858270
- [11] C. Liu, G. D. Clark, and J. Lindqvist, "Guessing attacks on usergenerated gesture passwords," *Proc. ACM Interact. Mob. Wearable Ubiquitous Technol.*, vol. 1, no. 1, pp. 3:1–3:24, Mar. 2017. [Online]. Available: http://doi.acm.org/10.1145/3053331
- [12] S. Komanduri, R. Shay, P. G. Kelley, M. L. Mazurek, L. Bauer, N. Christin, L. F. Cranor, and S. Egelman, "Of passwords and people: Measuring the effect of password-composition policies," in *Proceedings* of the SIGCHI Conference on Human Factors in Computing Systems, ser. CHI '11. New York, NY, USA: ACM, 2011, pp. 2595–2604. [Online]. Available: http://doi.acm.org/10.1145/1978942.1979321
- [13] J. Bonneau, S. Preibusch, and R. Anderson, "A birthday present every eleven wallets? the security of customer-chosen banking pins," in *Financial Cryptography and Data Security*, A. D. Keromytis, Ed. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012, pp. 25–40.
- [14] G. Cho, J. H. Huh, J. Cho, S. Oh, Y. Song, and H. Kim, "Syspal: Systemguided pattern locks for android," in 2017 IEEE Symposium on Security and Privacy (SP). IEEE, May 2017, pp. 338–356.
- [15] Y. Li, "Protractor: A fast and accurate gesture recognizer," in Proceedings of the SIGCHI Conference on Human Factors in Computing Systems, ser. CHI '10. New York, NY, USA: ACM, 2010, pp. 2169– 2172. [Online]. Available: http://doi.acm.org/10.1145/1753326.1753654

- [16] E. M. Taranta II, A. Samiei, M. Maghoumi, P. Khaloo, C. R. Pittman, and J. J. LaViola Jr., "Jackknife: A reliable recognizer with few samples and many modalities," in *Proceedings of the 2017 CHI Conference on Human Factors in Computing Systems*, ser. CHI '17. New York, NY, USA: ACM, 2017, pp. 5850–5861. [Online]. Available: http://doi.acm.org/10.1145/3025453.3026002
- [17] A. Sahami Shirazi, P. Moghadam, H. Ketabdar, and A. Schmidt, "Assessing the vulnerability of magnetic gestural authentication to video-based shoulder surfing attacks," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '12. New York, NY, USA: ACM, 2012, pp. 2045–2048. [Online]. Available: http://doi.acm.org/10.1145/2207676.2208352
- [18] A. De Luca, A. Hang, F. Brudy, C. Lindner, and H. Hussmann, "Touch me once and i know it's you!: Implicit authentication based on touch screen patterns," in *Proceedings of the SIGCHI Conference* on Human Factors in Computing Systems, ser. CHI '12. New York, NY, USA: ACM, 2012, pp. 987–996. [Online]. Available: http://doi.acm.org/10.1145/2207676.2208544
- [19] S. Komanduri, R. Shay, P. G. Kelley, M. L. Mazurek, L. Bauer, N. Christin, L. F. Cranor, and S. Egelman, "Of passwords and people: Measuring the effect of password-composition policies," in *Proceedings* of the SIGCHI Conference on Human Factors in Computing Systems, ser. CHI '11. New York, NY, USA: ACM, 2011, pp. 2595–2604. [Online]. Available: http://doi.acm.org/10.1145/1978942.1979321
- [20] R. Shay, S. Komanduri, P. G. Kelley, P. G. Leon, M. L. Mazurek, L. Bauer, N. Christin, and L. F. Cranor, "Encountering stronger password requirements: User attitudes and behaviors," in *Proceedings* of the Sixth Symposium on Usable Privacy and Security, ser. SOUPS '10. New York, NY, USA: ACM, 2010, pp. 2:1–2:20. [Online]. Available: http://doi.acm.org/10.1145/1837110.1837113
- [21] G. D. Clark, J. Lindqvist, and A. Oulasvirta, "Composition policies for gesture passwords: User choice, security, usability and memorability," in 2017 IEEE Conference on Communications and Network Security (CNS). IEEE, Oct 2017, pp. 1–9.
- [22] J. Ma, W. Yang, M. Luo, and N. Li, "A study of probabilistic password models," in 2014 IEEE Symposium on Security and Privacy, May 2014, pp. 689–704.
- [23] L. Anthony and J. O. Wobbrock, "\$n-protractor: A fast and accurate multistroke recognizer," in *Proceedings of Graphics Interface* 2012, ser. GI '12. Toronto, Ont., Canada, Canada: Canadian Information Processing Society, 2012, pp. 117–120. [Online]. Available: http://dl.acm.org/citation.cfm?id=2305276.2305296
- [24] D. H. Douglas and T. K. Peucker, "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature," *Cartographica: The International Journal for Geographic Information and Geovisualization*, vol. 10, no. 2, pp. 112–122, 1973. [Online]. Available: https://doi.org/10.3138/FM57-6770-U75U-7727
- [25] J. Bonneau, "The science of guessing: Analyzing an anonymized corpus of 70 million passwords," in *Proceedings of the 33rd IEEE Symposium* on Security and Privacy, May 2012, pp. 538–552.
- [26] B. J. Frey and D. Dueck, "Clustering by passing messages between data points," *Science*, vol. 315, no. 5814, pp. 972–976, 2007. [Online]. Available: http://science.sciencemag.org/content/315/5814/972
- [27] M. Harbach, A. De Luca, and S. Egelman, "The anatomy of smartphone unlocking: A field study of android lock screens," in *Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems*, ser. CHI '16. New York, NY, USA: ACM, 2016, pp. 4806–4817. [Online]. Available: http://doi.acm.org/10.1145/2858036.2858267
- [28] S. Cha, S. Kwag, H. Kim, and J. H. Huh, "Boosting the guessing attack performance on android lock patterns with smudge attacks," in *Proceedings of the 2017 ACM on Asia Conference on Computer and Communications Security*, ser. ASIA CCS '17. New York, NY, USA: ACM, 2017, pp. 313–326. [Online]. Available: http://doi.acm.org/10.1145/3052973.3052989
- [29] S. T. Haque, M. Wright, and S. Scielzo, "A study of user password strategy for multiple accounts," in *Proceedings of the Third ACM Conference on Data and Application Security and Privacy*, ser. CODASPY '13. New York, NY, USA: ACM, 2013, pp. 173–176. [Online]. Available: http://doi.acm.org/10.1145/2435349.2435373
- [30] M. J. Cortese and A. Fugett, "Imageability ratings for 3,000 monosyllabic words," *Behavior Research Methods, Instruments, & Computers*, vol. 36, no. 3, pp. 384–387, Aug 2004. [Online]. Available: https://doi.org/10.3758/BF03195585

- [31] J. Schock, M. J. Cortese, and M. M. Khanna, "Imageability estimates for 3,000 disyllabic words," *Behavior Research Methods*, vol. 44, no. 2, pp. 374–379, Jun 2012. [Online]. Available: https://doi.org/10.3758/s13428-011-0162-0
- [32] M. Brysbaert, A. B. Warriner, and V. Kuperman, "Concreteness ratings for 40 thousand generally known english word lemmas," *Behavior Research Methods*, vol. 46, no. 3, pp. 904–911, Sep 2014. [Online]. Available: https://doi.org/10.3758/s13428-013-0403-5
- [33] M. Brysbaeri, P. Mandera, S. F. McCormick, and E. Keuleers, "Word prevalence norms for 62,000 english lemmas," *Behavior Research Methods*, Jul 2018. [Online]. Available: https://doi.org/10. 3758/s13428-018-1077-9
- [34] V. Kuperman, H. Stadthagen-Gonzalez, and M. Brysbaert, "Age-ofacquisition ratings for 30,000 english words," *Behavior Research Methods*, vol. 44, no. 4, pp. 978–990, Dec 2012. [Online]. Available: https://doi.org/10.3758/s13428-012-0210-4
- [35] A. B. Warriner, V. Kuperman, and M. Brysbaert, "Norms of valence, arousal, and dominance for 13,915 english lemmas," *Behavior Research Methods*, vol. 45, no. 4, pp. 1191–1207, Dec 2013. [Online]. Available: https://doi.org/10.3758/s13428-012-0314-x
- [36] I. Oakley, J. H. Huh, J. Cho, G. Cho, R. Islam, and H. Kim, "The personal identification chord: A four buttonauthentication system for smartwatches," in *Proceedings of the 2018 on Asia Conference on Computer and Communications Security*, ser. ASIACCS '18. New York, NY, USA: ACM, 2018, pp. 75–87. [Online]. Available: http://doi.acm.org/10.1145/3196494.3196555
- [37] R. Shay, S. Komanduri, A. L. Durity, P. S. Huh, M. L. Mazurek, S. M. Segreti, B. Ur, L. Bauer, N. Christin, and L. F. Cranor, "Can long passwords be secure and usable?" in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, ser. CHI '14. New York, NY, USA: ACM, 2014, pp. 2927–2936. [Online]. Available: http://doi.acm.org/10.1145/2556288.2557377
- [38] L. Qiu, A. De Luca, I. Muslukhov, and K. Beznosov, "Towards understanding the link between age and smartphone authentication," in *Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems*, ser. CHI '19. New York, NY, USA: ACM, 2019, pp. 163:1–163:10. [Online]. Available: http://doi.acm.org/10. 1145/3290605.3300393

APPENDIX A

SUBJECTIVE CATEGORIZATION OF GESTURES

Table X reports subjective categorization rates for gestures. The first seven categories appear in prior work [11], while the remaining four categories of Compound, Cursive, Iconic and Other were newly introduced. Compound gestures are complex, involving at least two or more elements (e.g., a shape and a letter); cursive gestures involve short sequences of letters (e.g., hand-writing) and; iconic gestures depict objects (e.g., hands and flowers). Other collects any remaining gestures.

TABLE X

Mean subjective categorization rates from two independent raters of gestures in both studies. Categorizations used a random 10% of first study data (260 gestures) and a random 25% of each policy in the second study (250 gestures), samples selected to be numerically similar in size. Inter-rater agreement scores (Cohen's kappa) are shown in the final row. The rightmost column reports the mean categorization rates from both data sets in [11] for reference.

Digit 9.04% 11.20% 7.00% 12.60% 9.20% 9.25% Geometric Shape 22.50% 30.00% 33.80% 20.60% 11.60% 44.90% Letter 22.80% 21.80% 20.20% 11.60% 14.80% 28.45% Math Function 2.89% 3.20% 5.00% 6.60% 9.80% 9.10% Math Symbol 9.81% 2.80% 2.40% 4.80% 4.40% Mush Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.70%	[11]
Geometric Shape 22.50% 30.00% 33.80% 20.60% 11.60% 44.90% Letter 22.89% 21.80% 50.00% 6.60% 9.80% 9.10% Math Function 2.89% 3.20% 5.00% 6.60% 9.80% 9.10% Math Symbol 9.81% 2.80% 2.40% 4.40% 4.40% Music Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.70%	
Letter 22.89% 21.80% 20.20% 11.60% 14.80% 28.45 Math Function 2.89% 3.20% 5.00% 6.60% 9.80% 9.109 Math Symbol 9.81% 2.80% 2.40% 4.40% 4.40% Music Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.709	6
Math Function 2.89% 3.20% 5.00% 6.60% 9.80% 9.10% Math Symbol 9.81% 2.80% 2.40% 4.80% 4.40% 4.40% Music Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.70%	6
Math Symbol 9.81% 2.80% 2.40% 4.80% 4.40% 4.40% Music Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.70%	
Music Symbol 0.96% 0.20% 0.40% 0.00% 0.40% 2.70%	,
a 1 tan - a ann - 1 ann	,
Special Character 0.00% 4.80% 1.40% 2.40% 1.20% 2.20%	,
Compound 22.12% 16.60% 14.40% 35.20% 39.00% N/A	
Cursive 4.04% 2.80% 7.60% 3.00% 3.80% N/A	
Iconic 5.19% 6.20% 7.60% 3.00% 5.00% N/A	
Other 0.58% 0.40% 0.20% 0.20% 0.80% N/A	
$\kappa = 0.40$ $\kappa = 0.29$ $\kappa = 0.38$ $\kappa = 0.50$ $\kappa = 0.44$	
A construct $p < .001$ $p = .003$ $p < .001$ $p = .001$ N/A	
Agreement Moderate Fair Fair Moderate Moderate	

First study and second study baseline and lexical categorizations are broadly consistent. Second study blacklist and consolidated policies show reductions in gestures categorized as geometric shapes and letters and elevated rates for compound categorizations. Compared to prior work (right column), gesture categorizations show reduced rates for geometric shapes and letters due to the introduction of new categories (particularly compound) and a reduced occurrence of math and music symbols, possibly due to different populations (university students in [11] versus MTurk workers here).

APPENDIX B DISTRIBUTION OF GESTURE POINTS IN POLICY CONDITIONS IN FIRST AND SECOND STUDIES

Figure 9 depicts the distribution of gesture points in both studies. All results show bias in start and final point locations; first study data and the second study baseline policy are most strongly biased (to left corners) in start position and the lexical policy most evenly distributed in start and final positions.

	St	art Positi	on		Cen	ter Posit	ions		Final Position					
	28.4%	10.9%	8.5%		7.6%	12.7%	10.0%		6.2%	7.2%	16.1%			
Study 1	12.6%	6.8%	1.5%		11.3%	19.7%	10.4%		6.6%	9.3%	11.1%			
	23.8%	5.2%	2.4%		8.9%	11.2%	8.2%		13.3%	11.3%	18.9%			
	29.3%	10.0%	7.4%		8.1%	13.0%	10.1%		7.8%	5.6%	13.3%			
Study 2 Baseline	13.3%	9.1%	1.3%		11.3%	19.0%	9.8%		6.3%	11.4%	12.0%			
	23.5%	4.3%	1.8%		9.0%	10.8%	8.8%		13.1%	11.9%	18.6%			
	21.2%	15.6%	6.7%		6.1%	13.7%	7.1%		5.5%	11.4%	14.6%			
Study 2 Lexical	20.2%	9.2%	2.7%		13.9%	23.2%	11.9%		9.7%	11.9%	14.4%			
	16.8%	6.7%	0.9%		6.6%	11.7%	5.7%		7.3%	11.4%	13.8%			
	24.4%	15.5%	13.4%		7.5%	14.3%	8.8%		5.6%	8.7%	11.6%			
Study 2 Blacklist	10.0%	15.1%	2.0%		11.7%	21.9%	10.3%		5.5%	14.3%	14.1%			
	11.4%	5.6%	2.6%		6.8%	11.5%	7.2%		8.9%	12.2%	19.1%			
pa	27.8%	16.6%	8.9%		7.4%	13.7%	8.3%		7.2%	8.6%	11.5%			
Study 2 nsolidate	14.4%	11.5%	1.8%		11.8%	21.9%	10.4%		6.5%	13.7%	16.3%			
S	13.7%	3.3%	2.0%		7.5%	11.4%	7.5%		7.1%	10.0%	19.1%			

Fig. 9. Distribution of stroke points in user-chosen gestures in both studies. Columns depict start (left), final (right) and all other points (center) while rows depict data from first study (top) and second study baseline (center-top), lexical (center), blacklist (center-bottom) and consolidated (bottom).

Appendix C

n-gram Model Results

Table XI presents details of the 134 n-gram models generated from gestures captured in the first study that achieved a crack rate of greater than 10%. These data highlight the challenges in selecting appropriate n-gram models as high crack rates can be combined with low similarity to the original user-chosen gestures. Similarly, high similarity scores can be obtained with larger models (i.e., with more fine-grained discretizations) while the number of n-gram cases observed can be reduced.

TABLE XI

SUBSET OF 134 *n*-gram Models generated from first study data that achieved a crack rate of greater than 10%. Properties of Crack Rate (CR), Similarity (SM) and Completeness (CP) metrics are reported for each model. The selection models for Optimization are highlighted in **BOLD**.

In Aug Phase Smoothing Excl. CR SM CP Lan Aug Phase Smoothing Sole RS.5% 2 6 orbitet add-1 dad-1 <			Model Pa	odel Parameters Model Performance						Model Parameters					Model Performance			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Len	Ang	Phase	Smoothing	Excl.	CR	SM	CP	Len	Ang	Phase	Smoothing	Excl.	CR	SM	CP		
2 6 offset adal-1 dual 12.6% 45.3% 98.81% 3 10 offset adal-1 ingle 11.2% 73.9% 86.5% 2 6 offset adal-1/n adal-1/n ingle 12.9% 73.9% 86.5% 2 6 offset adal-1/n ingle 12.9% 73.9% 88.5% 2 6 offset adal-1/n ingle 12.9% 73.9% 88.5% 3 10 offset adal-1/n ingle 12.9% 85.3% 88.1% 3 10 adgeset 0.0% 70.5% 88.1% 2 6 offset add-1/n ingle 12.9% 85.3% 83.1% 3 12 offset add-1/n ingle 12.9% 76.5% 82.7% 2 8 offset add-1/n ingle 12.9% 76.5% 82.7% 2 8 afgeed add-1/n ingle 12.9%<	2	6	offset	add-1	all	12.99%	45.53%	98.81%	3	10	offset	add-1	single	11.95%	73.59%	88.65%		
2 6 offset add-1 single 12.95 73.59% 88.45% 2 6 offset add-1n single 12.95% 73.59% 88.45% 2 6 offset add-1n single 12.95% 73.59% 88.45% 2 6 offset add-1n single 12.95% 73.59% 88.15% 2 6 offset Good-Turing dall 12.95% 73.59% 88.15% 2 6 offset Good-Turing dall 12.95% 75.55% 88.15% 2 8 offset add-1 dual 12.95% 75.65% 82.75% 2 8 offset add-1 dual 16.95% 58.98% 95.85% 3 12 offset add-1 single 11.26% 75.65% 82.75% 2 8 offset add-1 dual 16.95% 58.98% 97.22% 3 12 offset	2	6	offset	add-1	dual	12.68%	45.53%	98.81%	3	10	offset	add-1/n	dual	15.23%	73.59%	88.65%		
2 0 offset Good Turing and L 1.9.9% 86.35% 98.81% 3 10 andfact andfact 1.9.9% 86.35% 98.81% 3 10 andfact andfact 1.9.9% 86.35% 98.81% 3 10 andfact andfact 1.9.9% 88.35% 88.31% 3 10 andfact andfact 1.9.9% 88.35% 88.31% 3 10 andfact andfact 1.9.9% 88.35% 88.31% 3 10 andfact andfact 1.9.9% 7.6.5% 82.35% 98.81% 3 12 offset andfact 1.9.9% 7.6.5% 82.35% 98.81% 3 12 offset andfact 1.9.9% 7.6.5% 82.35% 3 12 offset andfact 1.9.9% 7.6.5% 82.35% 3 12 offset andfact 1.9.9% 7.6.5% 82.35% 3 12 andfact andfact 1.9.9% 7.6.5% 82.35% 82.35% <t< td=""><td>2</td><td>6</td><td>offset</td><td>add-1</td><td>single</td><td>12.95%</td><td>45.53%</td><td>98.81%</td><td>3</td><td>10</td><td>offset</td><td>add-1/n</td><td>single</td><td>11.26%</td><td>73.59%</td><td>88.65%</td></t<>	2	6	offset	add-1	single	12.95%	45.53%	98.81%	3	10	offset	add-1/n	single	11.26%	73.59%	88.65%		
2 6 offset add-1n dual 12.7% 45.55% 98.81% 3 10 offset Good Turing add-1n dual 12.6% 73.5% 88.15% 2 6 offset Good Turing add-1 dual 14.6% 70.51% 88.15% 2 6 offset Good Turing single 12.6% 45.5% 89.81% 3 10 add-1n dual 14.6% 70.5% 82.7% 2 6 offset add-1 dual 10.2% 52.5% 89.8% 53.5% 3 12 offset add-1 dual 10.9% 76.5% 82.7% 2 8 offset add-1 single 12.3% 46.8% 97.22% 3 12 affget add-1 dual 12.9% 76.5% 82.7% 2 8 afgged add-1 single 12.3% 46.8% 97.22% 3 12 afgged add-1 <t< td=""><td>2</td><td>6</td><td>offset</td><td>add-1/n</td><td>all</td><td>12.99%</td><td>45.53%</td><td>98.81%</td><td>3</td><td>10</td><td>offset</td><td>Good-Turing</td><td>dual</td><td>15.23%</td><td>73.59%</td><td>88.65%</td></t<>	2	6	offset	add-1/n	all	12.99%	45.53%	98.81%	3	10	offset	Good-Turing	dual	15.23%	73.59%	88.65%		
2 0 0 0 algeed alg	2	6	offset	add-1/n	dual	12.72%	45.53%	98.81%	3	10	offset	Good-Turing	single	11.83%	73.59%	88.65%		
2 6 offset docs/lump all 1299 42.558 98.81% 3 10 aligned add-1 dual 11.20% 71.59% 83.75% 2 6 aligned Good-Turing single 11.30% 75.55% 98.81% 3 12 offset add-1 mingle 11.39% 75.65% 82.75% 2 8 offset add-1 dual 16.92% 53.98% 93.81% 3 12 offset add-1 mingle 10.25% 75.65% 82.75% 2 8 offset add-1 dual 16.92% 53.88% 97.22% 3 12 aligned add-1 dual 11.84% 73.05% 85.75% 2 8 aligned add-1 dual 11.84% 46.85% 97.22% 3 12 aligned add-1 dual 11.84% 73.05% 85.75% 2 18 aligned Good-Turing dual	2	6	offset	add-1/n	single	12.95%	45.53%	98.81%	3	10	aligned	add-1	dual	14.69%	70.51%	88.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	6	offset	Good-Turing	all	12.99%	45.53%	98.81%	3	10	aligned	add-1/n	dual	13.26%	70.51%	88.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	6	offset	Good-Turing	dual	12.68%	45.53%	98.81%	3	10	aligned	Good-Turing	dual	14.46%	70.51%	88.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	6	offset	Good-Turing	single	12.95%	45.53%	98.81%	3	12	offset	add-1	dual	13.80%	76.56%	82.75%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	6	aligned	Good-Turing	single	16.06%	42.25%	99.40%	3	12	offset	add-1	single	10.06%	76.56%	82.75%		
2 8 offset aligned construming and add-1 add add 10.598 33.898% 05.33% 1 1 construming add add add 11.598 75.56% 82.75% 2 8 aligned add-1 dual 11.298 65.85% 97.22% 3 12 aligned add-1 dual 11.24% 73.05% 85.75% 2 8 aligned add-1 dual 11.24% 73.05% 85.75% 2 8 aligned Good-Turing dual 11.05% 67.85% 97.22% 3 14 offset add-1 isingle 10.2% 78.10% 80.39% 2 8 aligned Good-Turing dual 10.5% 61.14% 94.55% 3 14 aligned add-1 dual 11.80% 77.49% 79.5% 2 10 offset Good-Turing dual 10.53% 77.4% 94.55% 3 14 aligned add-1 dual <td>2</td> <td>8</td> <td>offect</td> <td>add-1</td> <td>dual</td> <td>16.90%</td> <td>58.98%</td> <td>95.85%</td> <td>2</td> <td>12</td> <td>offect</td> <td>add-1/n</td> <td>dual</td> <td>10.92%</td> <td>76.56%</td> <td>82.15%</td>	2	8	offect	add-1	dual	16.90%	58.98%	95.85%	2	12	offect	add-1/n	dual	10.92%	76.56%	82.15%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	0	offset	Good Turing	dual	16.92%	58 080%	95.85%	2	12	offset	Good Turing	dual	12.00%	76.56%	82.15%		
2 8 aligned add-1 signed signed signed add-1 signed si	2	0	aligned	add 1	dual	10.92%	J6.90%	95.85%	2	12	offset	Good Turing	cingle	10.18%	76.56%	82.15%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	8	aligned	add 1	single	12.4970	40.887	07 22%	3	12	aligned	add 1	dual	11 840%	73.05%	85 75%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	8	aligned	add 1/n	dual	12.37%	40.887	07 22%	2	12	aligned	add 1/n	dual	11.34%	73.05%	85.75%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	8	aligned	add 1/n	single	11.49%	40.887	07 22%	3	12	aligned	Good Turing	dual	12 14%	73.05%	85.75%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	8	aligned	Good-Turing	dual	10.18%	46.88%	97.2270	3	14	offset	add-1	dual	14.80%	78.10%	80.30%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	8	aligned	Good-Turing	single	12.26%	46.88%	97.2270	3	14	offset	add-1	single	10.52%	78.10%	80.30%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	offset	add_1	dual	16.65%	61 14%	94 55%	3	14	offset	add_1/n	dual	12.95%	78.10%	80.30%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	offset	add-1	single	17 58%	61 14%	94 55%	3	14	offset	Good-Turing	dual	13.07%	78.10%	80.30%		
	2	10	offset	add-1/n	dual	16 54%	61 14%	94 55%	3	14	aligned	add-1	dual	11.80%	77 49%	79.65%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	offset	add-1/n	single	17.77%	61.14%	94.55%	3	14	aligned	add-1/n	dual	10.56%	77 49%	79.65%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	offset	Good-Turing	dual	16 54%	61 14%	94 55%	3	14	aligned	Good-Turing	dual	12.34%	77 49%	79.65%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	offset	Good-Turing	single	17 77%	61 14%	94 55%	4	6	offset	add-1	dual	14 11%	59 33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	aligned	add-1	dual	12.76%	57.75%	94.55%	4	6	offset	add-1	single	11.68%	59.33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	aligned	add-1/n	dual	13.03%	57.75%	94.55%	4	6	offset	add-1/n	dual	14.15%	59.33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	aligned	add-1/n	single	10.06%	57.75%	94.55%	4	6	offset	add-1/n	single	11.60%	59.33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	aligned	Good-Turing	dual	14.65%	57.75%	94.55%	4	6	offset	Good-Turing	dual	14.15%	59.33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	10	aligned	Good-Turing	single	10.45%	57.75%	94.55%	4	6	offset	Good-Turing	single	11.60%	59.33%	90.38%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	add-1	dual	14.15%	64.26%	92.63%	4	8	offset	add-1	dual	10.87%	77.06%	84.56%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	add-1	single	13.61%	64.26%	92.63%	4	8	offset	add-1/n	dual	11.33%	77.06%	84.56%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	add-1/n	dual	14.42%	64.26%	92.63%	4	8	offset	Good-Turing	dual	11.45%	77.06%	84.56%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	add-1/n	single	13.76%	64.26%	92.63%	4	8	aligned	add-1	single	10.79%	62.88%	86.21%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	Good-Turing	dual	14.34%	64.26%	92.63%	4	8	aligned	add-1/n	single	10.25%	62.88%	86.21%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	offset	Good-Turing	single	13.80%	64.26%	92.63%	4	8	aligned	Good-Turing	single	10.56%	62.88%	86.21%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	add-1	dual	12.84%	60.68%	93.11%	4	10	offset	add-1	dual	12.03%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	add-1	single	11.49%	60.68%	93.11%	4	10	offset	add-1	single	14.57%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	add-1/n	dual	12.41%	60.68%	93.11%	4	10	offset	add-1/n	dual	11.87%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	add-1/n	single	11.57%	60.68%	93.11%	4	10	offset	add-1/n	single	13.76%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	Good-Turing	dual	12.99%	60.68%	93.11%	4	10	offset	Good-Turing	dual	11.80%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12	aligned	Good-Turing	single	11.64%	60.68%	93.11%	4	10	offset	Good-Turing	single	14.15%	81.46%	80.24%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	14	offset	add-1	dual	12.57%	63.69%	91.19%	4	10	aligned	add-1	dual	12.72%	78.60%	80.65%		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	2	14	offset	add-1	single	13.22%	63.69%	91.19%	4	10	aligned	add-1/n	dual	10.53%	78.60%	80.65%		
214offsetadd-1/nsingle13.15%63.69%91.19%412offsetadd-1dual15.46%84.70%73.67%214offsetGood-Turingdual12.11%63.69%91.19%412offsetadd-1dual11.60%84.70%73.67%214alignedadd-1dual11.33%63.61%90.95%412offsetadd-1/nsingle11.29%84.70%73.67%214alignedadd-1dual11.26%63.61%90.95%412offsetGood-Turingdual14.00%84.70%73.67%214alignedadd-1/nsingle10.18%63.61%90.95%412offsetGood-Turingdual14.05%84.70%73.67%214alignedadd-1/nsingle10.68%63.61%90.95%412alignedadd-1dual11.91%82.81%75.13%36offsetadd-1dual13.38%53.16%94.72%412alignedadd-1/nsingle10.95%82.81%75.13%36offsetadd-1dual13.38%53.16%94.72%412alignedGood-Turingdual10.95%82.81%75.13%36offsetadd-1dual13.38%53.16%94.72%412alignedGood-Turingsingle1	2	14	offset	add-1/n	dual	12.07%	63.69%	91.19%	4	10	aligned	Good-Turing	dual	12.03%	78.60%	80.65%		
214offsetGood-Iuringdual12.11% 63.69% 91.19%412offsetadd-1single11.60% 84.70% 73.67% 214alignedadd-1dual11.33% 63.61% 90.95%412offsetadd-1/ndual12.72% 84.70% 73.67% 214alignedadd-1single10.18% 63.61% 90.95%412offsetadd-1/nsingle11.45% 84.70% 73.67% 214alignedadd-1/ndual11.26% 63.61% 90.95%412offsetGood-Turingdual14.00% 84.70% 73.67% 214alignedadd-1/nsingle10.16% 63.61% 90.95%412alignedadd-1dual11.91% 82.81% 75.13% 214alignedadd-1single10.06% 63.61% 90.95%412alignedadd-1dual11.91% 82.81% 75.13% 36offsetadd-1dual13.38% 53.16% 94.72% 412alignedadd-1/ndual11.30% 82.81% 75.13% 36offsetadd-1/ndual13.38% 53.16% 94.72% 412alignedadd-1/ndual10.87% 82.81% 75.13% 36offsetadd-1/ndual13.38% 53.16% 94.72% 412	2	14	offset	add-1/n	single	13.15%	63.69%	91.19%	4	12	offset	add-1	dual	15.46%	84.70%	73.67%		
214offsetGood-Luringsingle13.15% 63.69% 91.9% 412offsetadd-1/ndual 12.72% 84.70% 73.67% 214alignedadd-1dual11.33% 63.61% 90.95% 412offsetadd-1/ndual 11.29% 84.70% 73.67% 214alignedadd-1/ndual11.26% 63.61% 90.95% 412offsetGood-Turingdual 11.45% 84.70% 73.67% 214alignedadd-1/ndual11.26% 63.61% 90.95% 412alignedadd-1dual 11.91% 82.81% 75.13% 36offsetadd-1dual10.91\% 63.61% 90.95% 412alignedadd-1/ndual 11.91% 82.81% 75.13% 36offsetadd-1dual13.38% 53.16% 94.72% 412alignedadd-1/ndual 11.30% 82.81% 75.13% 36offsetadd-1/nsingle 11.84% 53.16% 94.72% 412alignedGood-Turingdual 10.95% 82.81% 75.13% 36offsetadd-1/nsingle 11.84% 53.16% 94.72% 412alignedGood-Turingsingle 10.95% 82.81% 75.13% 36offsetadd-1/nsingle 11.84% 53.16% <td>2</td> <td>14</td> <td>offset</td> <td>Good-Turing</td> <td>dual</td> <td>12.11%</td> <td>63.69%</td> <td>91.19%</td> <td>4</td> <td>12</td> <td>offset</td> <td>add-1</td> <td>single</td> <td>11.60%</td> <td>84.70%</td> <td>73.67%</td>	2	14	offset	Good-Turing	dual	12.11%	63.69%	91.19%	4	12	offset	add-1	single	11.60%	84.70%	73.67%		
214alignedadd-1dual11.33%63.61%90.95%412offsetGod-Turingfull11.25%84.70%73.67%214alignedadd-1/ndual11.26%63.61%90.95%412offsetGood-Turingdual14.00%84.70%73.67%214alignedadd-1/nsingle10.68%63.61%90.95%412offsetGood-Turingsingle11.03%82.81%75.13%214alignedGood-Turingdual10.91%63.61%90.95%412alignedadd-1dual11.91%82.81%75.13%36offsetadd-1dual10.91%63.61%94.72%412alignedadd-1/nsingle11.03%82.81%75.13%36offsetadd-1/nsingle11.84%53.16%94.72%412alignedGood-Turingdual10.95%82.81%75.13%36offsetadd-1/nsingle11.84%53.16%94.72%412alignedGood-Turingdual10.95%82.81%75.13%36offsetadd-1/nsingle11.84%53.16%94.72%412alignedGood-Turingsingle10.95%82.81%75.13%36offsetadd-1/nsingle11.84%53.16%94.72%414offsetadd-1	2	14	offset	Good-Turing	single	13.15%	63.69%	91.19%	4	12	offset	add-1/n	dual	12.72%	84.70%	73.67%		
214alignedadd-1single 10.18° 63.61° 90.95° 4 12 offsetGood-Turingdual 14.00° 84.70° 73.67° 214alignedadd-1/nsingle 11.26° 63.61° 90.95° 4 12 offsetGood-Turingsingle 11.45° 84.70° 73.67° 214alignedadd-1/nsingle 10.68° 63.61° 90.95° 4 12 alignedadd-1dual 11.91° 82.81° 75.13° 36offsetadd-1dual 10.91° 63.61° 90.95° 4 12 alignedadd-1dual 11.03° 82.81° 75.13° 36offsetadd-1dual 13.38° 53.16° 94.72° 4 12 alignedadd-1/ndual 10.95° 82.81° 75.13° 36offsetadd-1/ndual 13.38° 53.16° 94.72° 4 12 alignedGood-Turing 10.95° 82.81° 75.13° 36offsetGood-Turingsingle 11.84° 53.16° 94.72° 4 12 alignedGood-Turing 10.95° 82.81° 75.13° 36offsetGood-Turingsingle 11.84° 53.16° 94.72° 4 12 alignedGood-Turing 10.95° <td>2</td> <td>14</td> <td>aligned</td> <td>add-1</td> <td>dual</td> <td>11.33%</td> <td>63.61%</td> <td>90.95%</td> <td>4</td> <td>12</td> <td>onset</td> <td>add-1/n</td> <td>single</td> <td>11.29%</td> <td>84.70%</td> <td>/3.6/%</td>	2	14	aligned	add-1	dual	11.33%	63.61%	90.95%	4	12	onset	add-1/n	single	11.29%	84.70%	/3.6/%		
214alignedadd-1/ndual11.26%63.61%90.95%412offsetGood-1uringsingle11.45%84.70%75.13%214alignedadd-1/nsingle10.68%63.61%90.95%412alignedadd-1dual11.91%82.81%75.13%36offsetadd-1dual13.38%53.16%94.72%412alignedadd-1/ndual11.30%82.81%75.13%36offsetadd-1dual13.38%53.16%94.72%412alignedadd-1/ndual10.95%82.81%75.13%36offsetadd-1/ndual13.38%53.16%94.72%412alignedGood-Turingdual10.87%82.81%75.13%36offsetadd-1/nsingle11.84%53.16%94.72%412alignedGood-Turingsingle10.87%82.81%75.13%36offsetGood-Turingsingle11.84%53.16%94.72%412alignedGood-Turingsingle10.95%82.81%75.13%36offsetGood-Turingsingle11.84%53.16%94.72%414offsetadd-1dual12.95%86.39%69.74%38offsetadd-1dual14.96%70.70%89.74%414offsetadd-1dual <td>2</td> <td>14</td> <td>aligned</td> <td>add-1</td> <td>single</td> <td>10.18%</td> <td>63.61%</td> <td>90.95%</td> <td>4</td> <td>12</td> <td>offset</td> <td>Good-Turing</td> <td>dual</td> <td>14.00%</td> <td>84.70%</td> <td>73.67%</td>	2	14	aligned	add-1	single	10.18%	63.61%	90.95%	4	12	offset	Good-Turing	dual	14.00%	84.70%	73.67%		
214alignedadd-1/nsingle10.08% 05.01% 90.95% 412alignedadd-1dual11.91% 82.81% 75.13% 36offsetadd-1dual13.38% 53.16% 94.72% 412alignedadd-1/ndual 11.03% 82.81% 75.13% 36offsetadd-1/ndual13.38% 53.16% 94.72% 412alignedadd-1/nsingle 11.03% 82.81% 75.13% 36offsetadd-1/ndual13.38% 53.16% 94.72% 412alignedadd-1/nsingle 10.95% 82.81% 75.13% 36offsetadd-1/ndual13.38% 53.16% 94.72% 412alignedGood-Turingdual 10.95% 82.81% 75.13% 36offsetadd-1/nsingle 11.84% 53.16% 94.72% 412alignedGood-Turingsingle 10.95% 82.81% 75.13% 36offsetGood-Turingsingle 11.84% 53.16% 94.72% 414offsetadd-1dual 12.95% 86.39% 69.74% 38offsetadd-1dual 14.49% 70.70% 89.74% 414offsetadd-1/ndual 12.53% 86.39% 69.74% 38offsetadd-1/ndual 14.49% 70.70% <td< td=""><td>2</td><td>14</td><td>aligned</td><td>add-1/n</td><td>dual</td><td>11.26%</td><td>63.61%</td><td>90.95%</td><td>4</td><td>12</td><td>onset</td><td>Good-Turing</td><td>single</td><td>11.45%</td><td>84.70%</td><td>75.07%</td></td<>	2	14	aligned	add-1/n	dual	11.26%	63.61%	90.95%	4	12	onset	Good-Turing	single	11.45%	84.70%	75.07%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	14	aligned	add-1/n	single	10.68%	63.61%	90.95%	4	12	aligned	add-1	dual	11.91%	82.81%	75.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	14	angned	Good-Turing	dual	10.91%	52.160	90.95%	4	12	aligned	add-1	single	11.03%	82.81%	75.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	0	offset	add-1	dual	13.38%	53.10%	94.72%	4	12	aligned	add-1/n	dual	11.30%	82.81%	75.13%		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	3	6	offset	aud-1	single	12 2807	53.10% 53.16%	24.1270 01 720%	4	12	aligned	Good Turing	dual	10.95%	02.01% 82.81%	75 120		
3 6 offset add-1n single 11.84% 53.16% 94.72% 4 12 angned Good-Turing fill 12.95% 86.39% 69.74% 3 6 offset Good-Turing single 11.84% 53.16% 94.72% 4 14 offset add-1 dual 12.95% 86.39% 69.74% 3 6 offset Good-Turing single 11.84% 53.16% 94.72% 4 14 offset add-1 dual 12.95% 86.39% 69.74% 3 8 offset add-1 dual 14.49% 70.70% 89.74% 4 14 offset add-1/n dual 12.53% 86.39% 69.74% 3 8 offset Good-Turing dual 14.49% 70.70% 89.74% 4 14 offset add-1/n single 11.57% 86.39% 69.74% 3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing <t< td=""><td>2</td><td>6</td><td>offect</td><td>add 1/n</td><td>aingle</td><td>11.30%</td><td>52 160</td><td>94.72%</td><td>4</td><td>12</td><td>aligned</td><td>Good Turing</td><td>cinala</td><td>10.87%</td><td>02.01% 92.91%</td><td>75.13%</td></t<>	2	6	offect	add 1/n	aingle	11.30%	52 160	94.72%	4	12	aligned	Good Turing	cinala	10.87%	02.01% 92.91%	75.13%		
3 6 offset Good-Turing utal 15.36% 95.16% 94.72% 4 14 offset add-1 dtal 12.95% 86.39% 69.74% 3 8 offset Good-Turing single 11.84% 53.16% 94.72% 4 14 offset add-1 dtal 12.95% 86.39% 69.74% 3 8 offset add-1 dtal 14.49% 70.70% 89.74% 4 14 offset add-1/n dtal 12.53% 86.39% 69.74% 3 8 offset add-1 dtal 14.96% 70.70% 89.74% 4 14 offset add-1/n single 11.57% 86.39% 69.74% 3 8 offset Good-Turing dtal 14.88% 70.70% 89.74% 4 14 offset add-1/n single 11.57% 86.39% 69.74% 3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing singl	2	6	offeet	Good Turing	dual	12 2807	53.10%	04 720L	4	14	offect	add 1	dual	12.95%	02.0170 86.2007	60 740		
3 8 offset add-1 dual 14.4% 5.16% 9.74% 4 14 offset add-1 single 10.95% 86.39% 69.74% 3 8 offset add-1 dual 14.49% 70.70% 89.74% 4 14 offset add-1/n dual 12.53% 86.39% 69.74% 3 8 offset add-1/n dual 14.96% 70.70% 89.74% 4 14 offset add-1/n dual 12.53% 86.39% 69.74% 3 8 offset Good-Turing dual 14.49% 70.70% 89.74% 4 14 offset add-1/n dual 12.45% 86.39% 69.74% 3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing single 11.26% 85.89% 69.33% 3 8 aligned add-1/n single 10.45% 55.86% 92.31% 4 14 aligned add-1 single	2	6	offect	Good-Turing	single	13.30%	53.10% 53.160%	24.1270 01 7202	4	14	offect	add 1	single	12.95%	86 200%	60 710%		
3 8 offset add-1/n dual 14.9.9% 70.70% 89.74% 4 14 offset add-1/n dual 11.57% 86.39% 69.74% 3 8 offset add-1/n dual 14.88% 70.70% 89.74% 4 14 offset add-1/n single 11.57% 86.39% 69.74% 3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing single 11.25% 86.39% 69.74% 3 8 aligned add-1/n dual 11.68% 55.86% 92.31% 4 14 offset Good-Turing single 11.26% 85.89% 69.33% 3 8 aligned add-1/n single 10.45% 55.86% 92.31% 4 14 aligned add-1 dual 11.26% 85.89% 69.33% 3 8 aligned add-1/n single 10.45% 55.86% 92.31% 4 14 aligned add-1 <td< td=""><td>3</td><td>8</td><td>offset</td><td>add-1</td><td>dual</td><td>11.0470</td><td>70.70%</td><td>94.1270 80.71%</td><td>4</td><td>14</td><td>offset</td><td>aud-1/n</td><td>dual</td><td>10.95%</td><td>86 30%</td><td>60 7/07</td></td<>	3	8	offset	add-1	dual	11.0470	70.70%	94.1270 80.71%	4	14	offset	aud-1/n	dual	10.95%	86 30%	60 7/07		
3 8 offset Good-Turing dual 14.88% 70.70% 89.74% 4 14 offset Good-Turing dual 12.45% 86.39% 69.74% 3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing dual 11.24% 86.39% 69.74% 3 8 aligned add-1 dual 11.68% 55.86% 92.31% 4 14 offset Good-Turing single 11.72% 86.39% 69.74% 3 8 aligned add-1 dual 11.68% 55.86% 92.31% 4 14 offset Good-Turing single 11.72% 86.39% 69.33% 3 8 aligned add-1 single 10.45% 55.86% 92.31% 4 14 aligned add-1 single 10.14% 85.89% 69.33% 3 8 aligned Good-Turing single 10.45% 55.86% 92.31% 4 14 aligned add-1	2	8	offeet	add-1/n	dual	14.47/0	70.70%	89.74%	4	14	offeet	add-1/n	single	11 57%	86 30%	60 710		
3 8 aligned add-1 single 10.72% 55.86% 92.31% 4 14 offset Good-Turing single 11.72% 86.39% 69.74% 3 8 aligned add-1/n dual 11.68% 55.86% 92.31% 4 14 offset Good-Turing single 11.72% 86.39% 69.74% 3 8 aligned add-1/n dual 11.68% 55.86% 92.31% 4 14 aligned add-1 dual 11.26% 85.89% 69.33% 3 8 aligned Good-Turing single 10.45% 55.86% 92.31% 4 14 aligned add-1 single 10.14% 85.89% 69.33% 3 8 aligned Good-Turing single 10.95% 55.86% 92.31% 4 14 aligned add-1 single 10.14% 85.89% 69.33% 3 10 offset add-1 dual 15.27% 73.59% 88.65% 4 14 aligned add-1	3	8	offset	Good-Turing	dual	14 88%	70 70%	89 74%	4	14	offset	Good-Turing	dual	12.45%	86 39%	69 74%		
3 8 aligned add-1/n single 10/12% 55.86% 92.31% 4 14 aligned add-1 dual 11.26% 85.89% 69.33% 3 8 aligned add-1/n single 10.45% 55.86% 92.31% 4 14 aligned add-1 dual 11.26% 85.89% 69.33% 3 8 aligned Good-Turing single 10.45% 55.86% 92.31% 4 14 aligned add-1 single 10.14% 85.89% 69.33% 3 10 offset add-1 dual 15.27% 73.59% 88.65% 4 14 aligned Good-Turing dual 10.18% 85.89% 69.33%	3	8	aligned	add_1	single	10 72%	55.86%	92.31%	4	14	offset	Good-Turing	single	11 72%	86 39%	69 74%		
3 8 aligned add-1/n single 10.45% 55.86% 92.31% 4 14 aligned add-1 single 10.45% 85.89% 69.33% 3 8 aligned Good-Turing single 10.95% 55.86% 92.31% 4 14 aligned add-1 dual 10.06% 85.89% 69.33% 3 10 offset add-1 dual 15.27% 73.59% 88.65% 4 14 aligned Good-Turing dual 10.18% 85.89% 69.33%	3	8	aligned	add-1/n	dual	11.68%	55.86%	92.31%	4	14	aligned	add-1	dual	11.26%	85.89%	69.33%		
3 8 aligned Good-Turing single 10.95% 55.86% 92.31% 4 14 aligned add-1/m dual 10.06% 85.89% 69.33% 3 10 offset add-1 dual 15.27% 73.59% 88.65% 4 14 aligned Good-Turing dual 10.18% 85.89% 69.33%	3	8	aligned	add-1/n	single	10.45%	55.86%	92.31%	4	14	aligned	add-1	single	10.14%	85.89%	69.33%		
3 10 offset add-1 dual 15.27% 73.59% 88.65% 4 14 aligned Good-Turing dual 10.18% 85.89% 69.33%	3	8	aligned	Good-Turing	single	10.95%	55.86%	92.31%	4	14	aligned	add-1/n	dual	10.06%	85.89%	69.33%		
	3	10	offset	add-1	dual	15.27%	73.59%	88.65%	4	14	aligned	Good-Turing	dual	10.18%	85.89%	69.33%		

APPENDIX D *n*-gram Stroke Distributions in First Study

Figure 10 depicts the stroke distribution in the three *n*-gram models chosen in the first study. Bias in the start stroke direction is towards rightwards and downwards, while center strokes tend to be short and final strokes are more likely to involve rightward motion.



Fig. 10. Distribution of strokes in optimized 2x10 (top row), 3x10 (center row) and 4x12 (bottom row) *n*-gram models in the first study. Left column shows data from start strokes, right column shows data from final strokes and all other strokes are shown in the center column. Each figure is divided into the discretization regions available in the given *n*-gram model, with the frequency of sub-strokes observed in each region marked in % and by color. Boundary regions are shown to scale as grey areas for both length and angle. The proportion of strokes used in each region (black text) and boundary (grey text) is shown at the boundary (for angles) and the bottom of each diagram (for lengths).

APPENDIX E *n*-gram Stroke Distributions in Second Study

Figure 11 depicts stroke distribution in n-gram models generated for all four policies in the second study. Bias in the start stroke direction is greatest in baseline. Center strokes are short while final strokes tend to be rightward.



Fig. 11. Distribution of strokes in baseline (top row), lexical (center-top row) and blacklist (center-bottom row) and consolidated (bottom row) n-gram models in the second study. Each model uses the configuration of the optimized 3x10 model from the first study. Left column shows data from start strokes, right column shows data from final strokes and all other strokes are shown in the center column.